Analytical Evaluation of the Effect of Raman Amplifier Induced Crosstalk in a WDM System.

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Abstract – **An analytical approach is developed to evaluate the impact of crosstalk due to Stimulated Raman Scattering (SRS) in a Raman amplifier on a WDM transmission system. The expression of crosstalk due to Stimulated Raman Scattering (SRS) in a Wavelength Division Multiplexed (WDM) system and the signal to crosstalk ratio is derived. The analysis results in an analytical approach to evaluate the effect of SRS on the performance of a WDM system. The analysis is extended to find the amount of crosstalk due to SRS and the derivation of bit error rate (BER) including the effect of crosstalk. The performance results are numerically evaluated in terms of BER as a function of different system parameters like received optical power, number of channels, channel spacing etc. The power penalty suffered by the system due to SRS induced crosstalk is also determined.**

Keyword – **SRS, WDM, Raman amplifier.**

I. Introduction

Stimulated Raman Scattering (SRS) is the inelastic scattering of a photon that produces a high frequency optical phonon. In Wavelength Division Multiplexed (WDM) system SRS is considered to be the ultimate limitation. In SRS, there is an interaction between light and vibrations of molecules which causes frequency conversion of light and results in excess attenuation of short-wavelength channels, i.e., Power depletion in WDM system. Thus producing signal crosstalk that degrades the system performance.

Analysis is carried out on SRS generated crosstalk in a WDM system along with the statistics of signal modulation to evaluate the penalty due to that crosstalk [1]. A number of oapproximate integrals and Gaussian approximation are used to derive the bit-error probability using optical amplifiers in a lightwave communication system. [2]. In WDM system the SRS crosstalk is evaluated for getting an analytical solution employing the triangular approximation to the spectrum of Raman gain[3]. In this paper, we present an analytical approach to evaluate the effect of Raman amplifier induced crosstalk in terms of bit error rate performance degradation and receiver sensitivity degradation with number of WDM channels. The power penalty suffered by a WDM system due to Raman amplifier induced crosstalk is also presented for several system parameters.

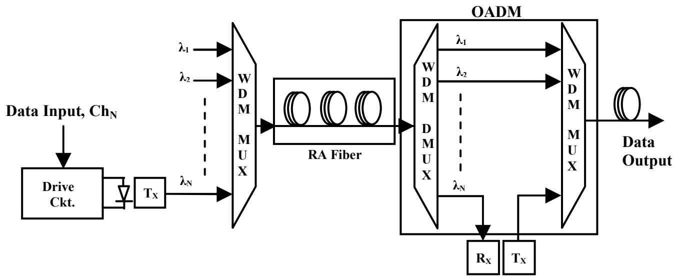

Figure 1. System Model of a WDM link with Raman Amplifier.

II. System Model

The WDM transmission system model used for theoretical analysis is shown in Figure 1 with Fiber Raman Amplifier as line amplifier. N data channels are used to modulate N laser sources of wavelengths $\lambda_1 \cdots \lambda_N$ using intensity modulation and are multiplexed by a WDM multiplexer. The output of the WDM multiplexer is fed to Fiber Raman Amplifier. The output of the amplifier is de-multiplexed and individual WDM channels are received by separate direct detection receivers in an Optical Add Drop Multiplexer (OADM). The output of the OADM is again passed to the fiber for further transmission.

III. Theoretical Analysis

We consider an IM/DD WDM system with N number of equally spaced channels in which the signals propagate at the same speed through the fiber. We evaluate the effect of SRS induced crosstalk as the depletion of power of the shortest wavelength channel, say channel-1 due to the (N-1) channel. The expression of the signal at the output of the WDM mux is given by:

$$E(t) = \sqrt{2P_0}\sum_{i=1}^{N} e^{j\left[\frac{2\pi c}{\lambda_i}t\right]} \tag{1}$$

where P_0 is the transmitted peak power of each channels and λ_i is the wavelength of the i-th WDM channel. The optical signal at the output of the Raman amplifier can be expressed as:

$$E_0(t) = \sqrt{2P_0}\sum_{i=1}^{N} e^{j\left[\frac{2\pi c}{\lambda_i}t\right]} e^{-\alpha L} e^{\gamma_i L} + \sum_{i=1}^{N} E_{c_i}(t) \tag{2}$$

where $E_c(t)$ denotes the crosstalk signal generated due to SRS process in the Fiber Raman amplifier, α is the fiber loss coefficient and γ is fiber gain coefficient, and the crosstalk signal cross ponding to i-th channel can be expressed as:

$$E_{c_i}(t) = \left[\sum_{i=2}^{N} \frac{P_0 \gamma_i L_{effi}}{2A_{effi}} .m_i\right] e^{j\left[\frac{2\pi c}{\lambda_i}t\right]} \quad (3)$$

The power depletion of the fiber channel is random which may be approximated by a Gaussian distribution with a mean η_i and variance $\sigma_i^2$for the i-th channel [1]. The total crosstalk is also considered Gaussian with mean and variance as the sum of the individual mean of variance of crosstalk due to individual channels.

The fractional power loss happened in the first channel can be derived as follows [1]:

$$\sum_{i=2}^{N} \frac{P_0 \gamma_i L_{eff}}{2A_{eff}} m_i \quad (4)$$

where P_0 is the peak power transmitted per channel, A_{eff} is the effective core area, L_{eff} is the effective fiber length.

$$L_{eff} = (1 - \exp(-\alpha L))/\alpha \quad (5)$$

α is the fiber loss coefficient and L is the fiber length. m_i is the modulation in the ith channel and can be assume the value 0 or 1 with the same probability1/2.γ_i is the Raman gain coefficient coupling the initial and ith channel . Considering the contribution of each channel the mean η or average optical intensity I_0 can be derived by the sum of the means η_i following ref.[1] as:

$$I_0 = \eta = \sum_{i=1}^{N-1} \eta_i = \sum_{i=1}^{N-1} \frac{i}{2} K = \frac{N(N-1)}{4} K \quad (6)$$

where

$$K = \frac{P \Delta f \gamma_p L_{eff}}{3 \times 10^{13} A_{eff}} 10 \log_{10} e$$

and γ_p is the peak Raman gain coefficient and Δf is the channel spacing.

The current due to signal at the photo detector output is given by:

$$i_d(t) = R_d |E_0(t)|^2 + i_n(t) \quad (7)$$

where R_d is the photo detector responsibility and $i_n(t)$ denotes the receiver thermal noise process.

The output photo detector current can be expressed as:

$$i_d(t) = R_d .P_0 .e^{-\alpha L} .e^{\gamma_p L} + i_c(t) + i_n(t) \quad (8)$$

where $i_c(t)$ is the current due to crosstalk.

The square crosstalk current can then be obtained as:

$$\sigma_c^2 = E[i_c^2(t)] == \sum_{i=1}^{N-1} \sigma_i^2 = \sum_{i=1}^{N-1} \frac{i^2}{4} K^2$$
$$= \frac{N(N-1)(2N-1)}{24} K^2 \quad (9)$$

and the mean signal current is given by

$$I_s = R_d .P_s \quad (10)$$

Where

$$P_s = P_T .e^{-\alpha L} .e^{\gamma_p L}$$

The signal to crosstalk plus noise ratio can be expressed as:

$$SCNR = \frac{RdPs}{\sqrt{\sigma_{th}^2 + \sigma_c^2}} = \frac{I_s}{\sigma_0} \quad (11)$$

where $\sigma_{th}^2 = 4KTB / R_L$ is the thermal noise current variance, R_L is the receiver load resistance, B is the receiver bandwidth, T is the receiver noise temperature.

Following ref. [2], the bit error rate can then be expressed as:

$$BER = \frac{\exp(-pTB_{opt}u)}{4\sqrt{\pi pTB_{opt}u}} . \left[1 + \frac{\sqrt{s} - 3\sqrt{s_d - 1}}{4pTB_{opt}u\sqrt{s_d - 1}}\right] \left(\frac{s}{s_d - 1}\right)^{1/4}$$
$$+ \frac{(pTB_{opt}s_d)^{pTB_{opt}-1}}{2\Gamma(pTB_{opt})} \left[1 + \frac{pTB_{opt} - 1}{pTB_{opt}} \frac{1}{s_d}\right] .\exp(-pTB_{opt}s_d) \quad (12)$$

where, $S_d = \frac{I_{th}}{I_0}$ and $u = \left(\sqrt{s} - \sqrt{s_d - 1}\right)^2, S = \frac{I_s}{\sigma_0}$

IV. RESULTS AND DISCUSSION

Following the analytical approach, the amount of crosstalk are evaluated for different number of channels and fiber lengths to observe the actual effect of Raman amplifier induced crosstalk in a WDM IM/DD system.

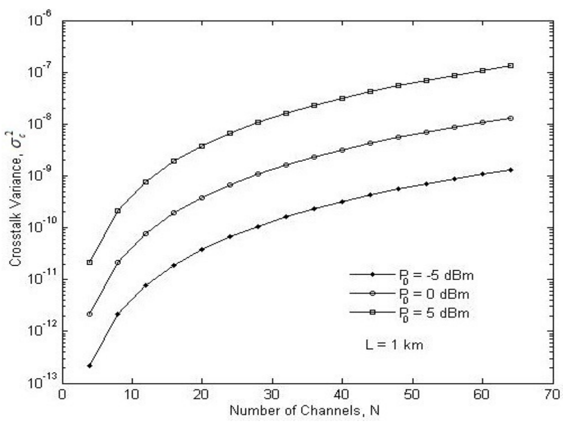

Figure 2. Crosstalk Variance, σ_c^2 versus Number of Channels, N for different values of Peak power, P_0 and length L=1 km.

The plots of crosstalk variance due to SRS are shown in Figure 2 as a function of number of WDM channels with peak power as a parameter and fiber length of 1 km. It is noticed that the amount of crosstalk is significants and is significantly large at larger number of WDM channels.

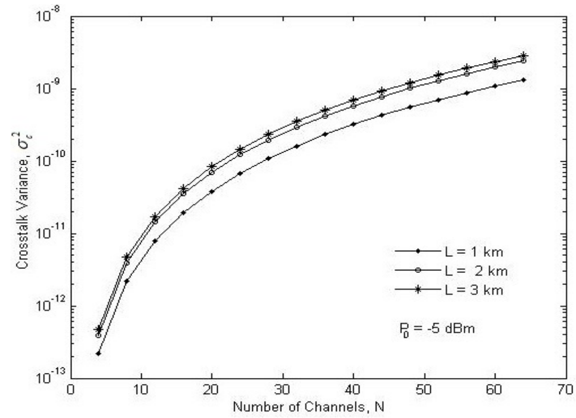

Figure 3. Plots of Crosstalk Variance, σ_c^2 versus Number of Channels, N for different lengths at Peak power, P_0 = -5 dBm.

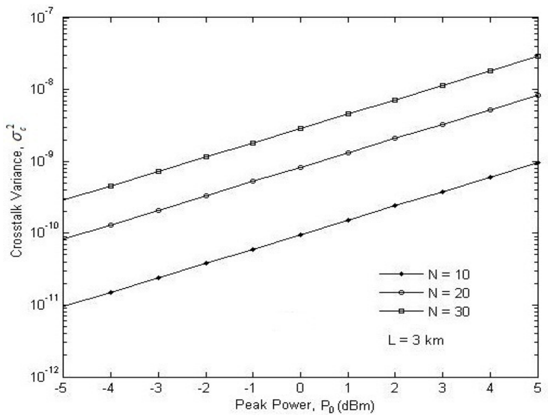

Figure 4. Plots Crosstalk Variance, σ_c^2 versus Peak Power, P_0 (dBm) for different number of Channels, N at length, L= 3 km.

The plots of crosstalk variance with fiber length as a parameter are shown in Figure 3 which indicates higher crosstalk at higher fiber length.

In Figure 4 the plots of Crosstalk Variance, σ_c^2 are shown as a function of peak power, P_0 (dBm) with different number of WDM channels, at a particular length,

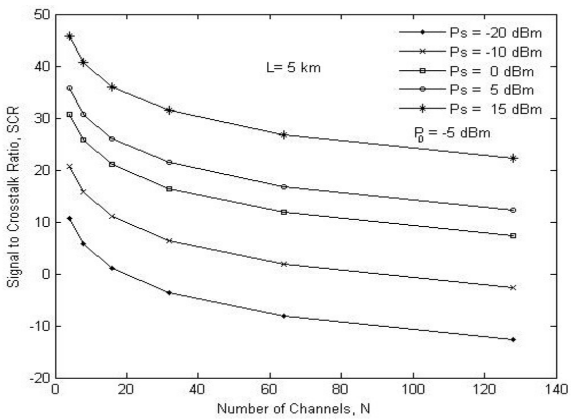

Figure 5. Plots of SCR versus Number of Channels, N for different values of Ps and P_0 = -5 dBm.

where it is noticed that there is increased crosstalk for increased peak power.

The variation of signal to crosstalk ratio (SCR) as a function of number of WDM channels is depicted in Figure 5 with signal power, P_s as a parameter. It is noticed that the SCR decrease with increase in the number of channels.

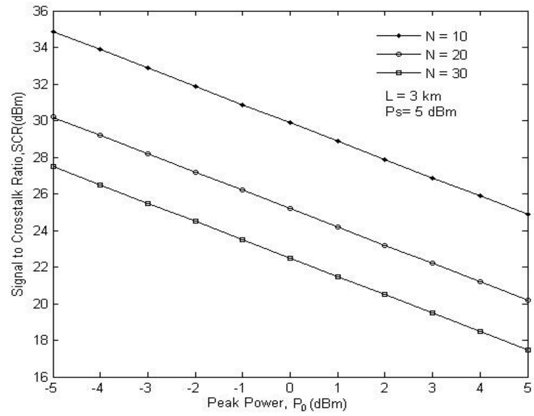

Figure 6. Plots of Signal to Crosstalk Ratio,SCR (dBm) versus Peak Power, P_0 for different Number of channels, N at length, L= 3 km and P_S = 5 dBm.

In Figure 6 another kind of variation of SCR as a function of peak power P_0 (dBm) is noticed with different number of WDM channels as a parameter at a certain fiber length and signal power P_S. In this figure, it is noticed that the SCR decreases with the increase in the peak power, P_0.

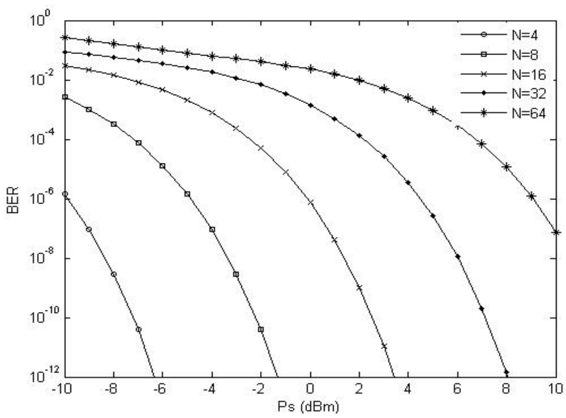

Figure 7. Plots of BER versus Ps (dBm) for different values of Channels, N and Normalized optical filter bandwidth, M = 1 (where $M = B_0/R_b$).

The plots of bit error rate versus received optical power Ps are depicted in Figure 7 for several values of the number of WDM channels, N. It is noticed that there is significant deterioration in BER performance with increase in the number of channels due to increased crosstalk. The required receiver sensitivity Ps(dBm) to achieve a BER of 10^{-9} are shown in Figure 8 as a function of number of WDM channels and is found to be higher at higher number of channels.

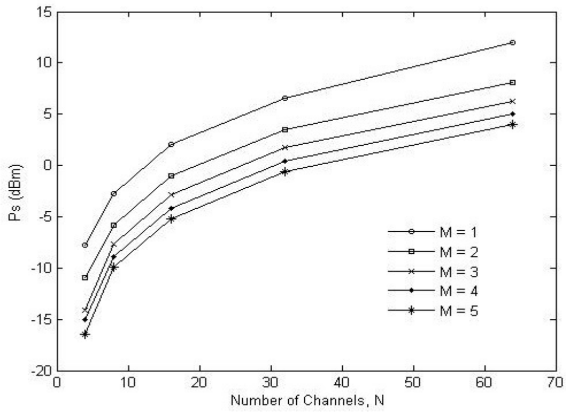

Figure 8. Plots of *Ps* (dBm) versus Number of Channels, *N* for different values of *M*

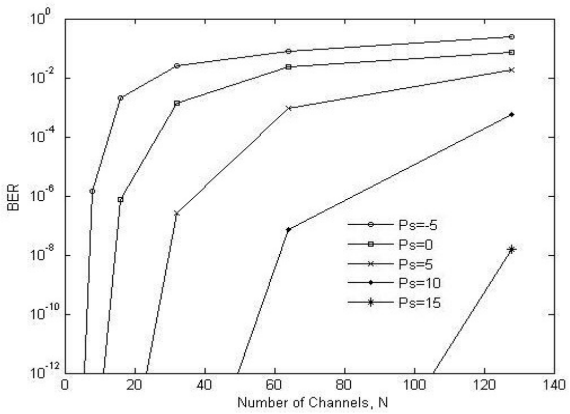

Figure 9. Plots of BER versus Number of Channels, *N* for different values of *Ps* and Normalized optical filter bandwidth, *M* = 1.

The variation of BER with number of WDM channels is shown in Figure 9 for different received optical signal level. As is evident from the figure, the BER increases with N at a given received optical power. At a BER of 10^{-9}, the allowable number of WDM channels can be found from this figure corresponding to a given value of *Ps*.

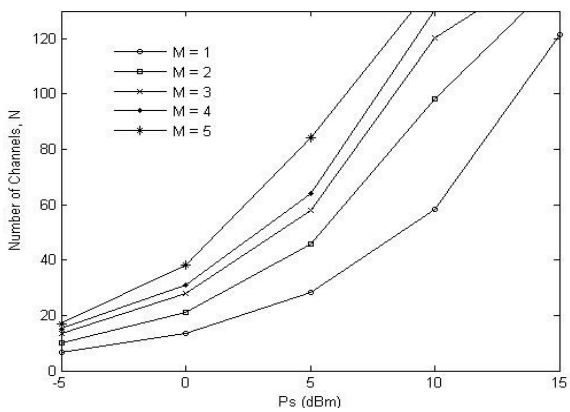

Figure 10. Plots of Number of Channels, *N* versus *Ps* (dBm) for different values of *M*.

The plots of allowable number of WDM channels corresponding to a BER of 10^{-9} are shown in Figure 10 as a function of receiver sensitivity *Ps*(dBm) with normalized optical filter bandwidth M as a parameter. It is observed that higher number of WDM channels requires a higher amount of received optical power to achieve a given BER at a given optical filter bandwidth. At increased optical bandwidth, there is additional power requirement as a power penalty due to crosstalk to achieve the same BER.

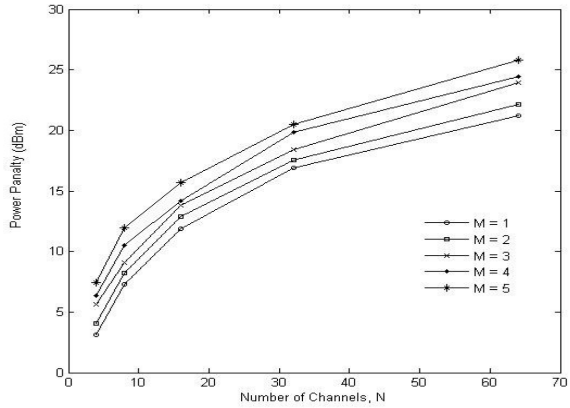

Figure 11. Plots of Power Penalty (dBm) due to SRS induced crosstalk versus Number of Channels, *N*.

Plots of Power Penalty due to SRS induced crosstalk are shown in Figure 11. Penalty is evaluated considering *N=2* as the reference. The plots are presented for different values of the optical bandwidth expansion factor, *M*. It is noticed that there are significant power penalty as the number of WDM channels are increased and at higher bandwidth.

V. Conclusions

An analytical approach is presented to evaluate the impact of Raman amplifier induced crosstalk in a WDM transmission system. The bit error rate performance results are evaluated numerically and the effect of crosstalk is evaluated in terms of degradation in receiver sensitivity at a given BER. The power penalty is found to be significant at increased value of number of WDM channels. The analytical approach will find its application evaluating the limitations of crosstalk in a WDM system.

References

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