

Generalized Mutually Orthogonal ZCZ Sequence Sets Based on Perfect Sequences and Orthogonal Codes

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Abstract—The present paper proposes a new method for constructing mutually orthogonal zero-correlation zone (MO-ZCZ) sequence sets using perfect sequences, orthogonal codes, and an interleaving technique. Generally, an MO-ZCZ sequence set is composed of several ZCZ sequence sets, and two arbitrary sequences that belong to different ZCZ sequence sets have orthogonality. Each ZCZ sequence set included in the proposed MO-ZCZ sequence set is a quasi-optimal ZCZ sequence set. In addition, two arbitrary sequences that belong to different ZCZ sequence sets have not only orthogonality but also a zero-correlation zone of which the ZCZ length satisfies quasi-optimality. Consequently, the proposed MO-ZCZ sequence set can be regarded as a single quasi-optimal ZCZ sequence set. In this sense, the proposed MO-ZCZ sequence set can be regarded as a generalized version of conventional MO-ZCZ sequence sets.

Keywords—MO-ZCZ sequence sets, Quasi-optimal ZCZ sequence sets, Spreading sequences, AS-CDMA systems, Spread spectrum communication

I. INTRODUCTION

Code-division multiple-access (CDMA) has been widely applied in digital cellular systems. In CDMA systems, channel separation is provided by the correlation properties of pseudo-random codes referred to as spreading sequences. Therefore, spreading sequences with good autocorrelation and cross-correlation properties play an important part in CDMA systems.

In recent years, approximately synchronized CDMA (AS-CDMA) systems have attracted a great deal of attention because co-channel interference within a cell does not exist in some types of AS-CDMA systems [1]. In such AS-CDMA systems, zero-correlation zone (ZCZ) sequence sets are used as spreading sequences in order to realize this advantage. Various types of ZCZ sequence sets have been widely studied [1]–[23]. Generally, a ZCZ sequence set is characterized by the sequence period, the number of sequences, the ZCZ length, and the number of phases of the sequence elements. The ZCZ length is restricted by a mathematical upper bound determined by the sequence period and the number of sequences [6], [24],

and ZCZ sequence sets that satisfy this mathematical upper bound are referred to as optimal ZCZ sequence sets. A number of studies have evaluated optimal ZCZ sequence sets [1], [6], [12], [16], [19], [22]. However, the methods proposed in these studies require certain conditions in order to construct optimal ZCZ sequence sets. As such, quasi-optimal ZCZ sequence sets have also been investigated extensively [9], [15], [23]. Quasi-optimal ZCZ sequence sets have a ZCZ length that is equal to one less than the mathematical upper bound. These methods can generate quasi-optimal ZCZ sequence sets even under conditions whereby no optimal ZCZ sequence sets exist.

Recently, as one type of ZCZ sequence set, mutually orthogonal ZCZ (MO-ZCZ) sequence sets have been proposed by some researchers [25]–[27]. As mentioned above, the ZCZ length is restricted by the sequence period and the number of sequences. This means that the number of sequences in a ZCZ sequence set is also restricted by the sequence period and the ZCZ length. MO-ZCZ sequence sets have been studied in order to increase the number of available sequences for AS-CDMA systems. An MO-ZCZ sequence set is composed of several ZCZ sequence sets, and two arbitrary sequences that belong to different ZCZ sequence sets have orthogonality.

In the present paper, we propose a new method for constructing MO-ZCZ sequence sets using perfect sequences, orthogonal codes, and an interleaving technique. Each ZCZ sequence set of the proposed MO-ZCZ sequence set becomes a quasi-optimal ZCZ sequence set. In addition, two arbitrary sequences that belong to different ZCZ sequence sets have not only orthogonality but also a zero-correlation zone of which the ZCZ length satisfies quasi-optimality. Consequently, the proposed MO-ZCZ sequence set can be regarded as a single quasi-optimal ZCZ sequence set. In this sense, the proposed MO-ZCZ sequence set can be regarded as a generalized version of conventional MO-ZCZ sequence sets and is more suitable for reducing co-channel interference than them.

II. PRELIMINARIES

In this section, we define ZCZ sequence sets, MO-ZCZ sequence sets, and related terms.

First, we explain ZCZ sequence sets. Let Z be a sequence set with N sequences of period P . Then, Z can be represented as

$$Z = \{Z_n \mid 0 \leq n \leq N-1\},$$

$$Z_n = (z_0^{(n)}, z_1^{(n)}, \dots, z_p^{(n)}, \dots, z_{P-1}^{(n)}), \quad (1)$$

where Z_n and $z_p^{(n)}$ represent a sequence and a sequence element, respectively. Let $R_{Z_{n_0}, Z_{n_1}}(\tau)$ be the periodic correlation function between Z_{n_0} and Z_{n_1} . For the sake of simplicity, the modulo operator is represented as $\%P$, i.e.,

$$x \% P \stackrel{\text{def}}{=} x \bmod P. \quad (2)$$

Then, $R_{Z_{n_0}, Z_{n_1}}(\tau)$ is defined as

$$R_{Z_{n_0}, Z_{n_1}}(\tau) \stackrel{\text{def}}{=} \sum_{p=0}^{P-1} z_p^{(n_0)} z_{(p+\tau)\%P}^{(n_1)*}, \quad (3)$$

where the symbol $*$ denotes complex conjugation. When Z_{n_0} does not correspond to Z_{n_1} , $R_{Z_{n_0}, Z_{n_1}}(\tau)$ is referred to as the periodic cross-correlation function between Z_{n_0} and Z_{n_1} . On the other hand, when Z_{n_0} corresponds to Z_{n_1} , $R_{Z_{n_0}, Z_{n_1}}(\tau) = R_{Z_{n_0}, Z_{n_0}}(\tau)$ is referred to as the periodic autocorrelation function of Z_{n_0} . If all of the sequences in Z satisfy the following conditions, Z is referred to as a ZCZ sequence set:

$$\forall n_0, 1 \leq |\tau| \leq L,$$

$$R_{Z_{n_0}, Z_{n_0}}(\tau) = 0, \quad (4)$$

$$\forall n_0 \neq n_1, |\tau| \leq L,$$

$$R_{Z_{n_0}, Z_{n_1}}(\tau) = 0. \quad (5)$$

The integer L is referred to as a ZCZ length. The ZCZ sequence set is represented as $Z_{cz}(P, N, L)$ in order to illustrate the sequence period, the number of sequences, and the ZCZ length. The ZCZ length is restricted by the following mathematical upper bound [6], [24]:

$$L \leq \frac{P}{N} - 1. \quad (6)$$

If a ZCZ sequence set satisfies the following condition, the ZCZ sequence set is referred to as an optimal ZCZ sequence set:

$$L = \frac{P}{N} - 1. \quad (7)$$

Similarly, a quasi-optimal ZCZ sequence set is defined as a ZCZ sequence set that satisfies the following condition:

$$L = \frac{P}{N} - 2. \quad (8)$$

Next, we explain MO-ZCZ sequence sets. Let M be a set with K ZCZ sequence sets containing N sequences of period

P . Then, M can be represented as

$$M = \{M_k \mid 0 \leq k \leq K-1\},$$

$$M_k = \{M_n^{(k)} \mid 0 \leq n \leq N-1\},$$

$$M_n^{(k)} = (m_0^{(k,n)}, \dots, m_p^{(k,n)}, \dots, m_{P-1}^{(k,n)}), \quad (9)$$

where M_k , $M_n^{(k)}$, and $m_p^{(k,n)}$ represent a ZCZ sequence set, a sequence, and a sequence element, respectively. If two arbitrary sequences that belong to different ZCZ sequence sets satisfy the following condition, M is referred to as an MO-ZCZ sequence set:

$$\forall k_0 \neq k_1, \forall n_0, \forall n_1,$$

$$R_{M_{n_0}^{(k_0)}, M_{n_1}^{(k_1)}}(0) = 0. \quad (10)$$

That is, an MO-ZCZ sequence set is a set of several ZCZ sequence sets, and two arbitrary sequences that belong to different ZCZ sequence sets have orthogonality. From (6), it is obvious that the number of sequences in a ZCZ sequence set is also restricted by the following mathematical upper bound:

$$N \leq \frac{P}{L+1}. \quad (11)$$

MO-ZCZ sequence sets have been studied in order to increase the number of available sequences for AS-CDMA systems.

III. NEW MO-ZCZ SEQUENCE SETS

In this section, we propose a new method for constructing MO-ZCZ sequence sets based on perfect sequences, orthogonal codes, and an interleaving technique.

Let $S = (s_0, s_1, \dots, s_p, \dots, s_{P-1})$ be a perfect sequence of period P , i.e., S satisfies the following autocorrelation property:

$$\text{For } \tau \neq 0,$$

$$R_{S,S}(\tau) = 0, \quad (12)$$

$$\text{For } \tau = 0,$$

$$R_{S,S}(\tau) = E_S, \quad (13)$$

where E_S is the sequence energy of S . Let N_0 be a positive integer, and assume that $N_0 \geq 2$ and $N_0 | P$. Let O be a set of N_0 orthogonal codes of length N_0 , i.e., O can be represented as

$$O = \{O_n \mid 0 \leq n \leq N_0-1\},$$

$$O_n = (o_0^{(n)}, o_1^{(n)}, \dots, o_p^{(n)}, \dots, o_{N_0-1}^{(n)}), \quad (14)$$

$$\forall n_0 \neq n_1,$$

$$R_{O_{n_0}, O_{n_1}}(0) = \sum_{p=0}^{N_0-1} o_p^{(n_0)} o_p^{(n_1)*} = 0, \quad (15)$$

where O_n and $o_p^{(n)}$ represent a sequence and a sequence element, respectively. Two integers N_1 and N_2 are defined as follows:

$$P = N_0 N_1 N_2, \quad (16)$$

$$N_1 \geq 2, N_2 \geq 1. \quad (17)$$

Using S and O , an MO-ZCZ sequence set M is obtained by the following formula:

$$\begin{aligned} M &= \{M_k \mid 0 \leq k \leq N_1 - 1\}, \\ M_k &= \{M_n^{(k)} \mid 0 \leq n \leq N_0 - 1\}, \\ M_n^{(k)} &= (m_0^{(k,n)}, m_1^{(k,n)}, \dots, m_p^{(k,n)}, \dots, m_{PN_0-1}^{(k,n)}), \\ m_p^{(k,n)} &= s((k+(p\%N_0)N_1)N_2 + \lfloor p/N_0 \rfloor \% P \cdot o_{p\%N_0}^{(n)}), \end{aligned} \quad (18)$$

where $\lfloor p/N_0 \rfloor$ is the floor function of p/N_0 . Note that M includes N_1 ZCZ sequence sets, each ZCZ sequence set includes N_0 sequences, and the period of each sequence is PN_0 . Each M_k becomes a ZCZ sequence set that can be represented as $Z_{cz}(PN_0, N_0, P-2)$. Since the parameters of this ZCZ sequence set satisfy (8), each M_k is a quasi-optimal ZCZ sequence set. Now, suppose that M is a single sequence set. Since M includes N_1 ZCZ sequence sets and each M_k includes N_0 sequences, the total number of sequences included in M is N_0N_1 . In this case, M becomes a ZCZ sequence set that can be represented as $Z_{cz}(PN_0, N_0N_1, P/N_1-2)$. Since the parameters of this ZCZ sequence set satisfy (8), M is also a quasi-optimal ZCZ sequence set. It is clear that two arbitrary sequences that belong to different ZCZ sequence sets have orthogonality because they have a zero-correlation zone. Therefore, M is an MO-ZCZ sequence set. Note that two arbitrary sequences that belong to different ZCZ sequence sets have not only orthogonality but also a zero-correlation zone. In this sense, the proposed MO-ZCZ sequence set can be regarded as a generalized version of conventional MO-ZCZ sequence sets.

IV. PROOF

In this section, we prove the proposed method.

Integers $p, p_0, p_1, \tau, \tau_0$, and τ_1 are defined as

$$p = p_1N_0 + p_0, \quad (19)$$

$$\tau = \tau_1N_0 + \tau_0, \quad (20)$$

$$0 \leq p_0, \tau_0 \leq N_0 - 1, \quad (21)$$

$$0 \leq p_1, \tau_1 \leq P - 1, \quad (22)$$

$$0 \leq p, \tau \leq PN_0 - 1. \quad (23)$$

In addition, an integer ϵ is defined as follows:

$$\begin{aligned} \text{For } p_0 + \tau_0 < N_0, \\ \epsilon &= 0, \end{aligned} \quad (24)$$

$$\begin{aligned} \text{For } p_0 + \tau_0 \geq N_0, \\ \epsilon &= 1. \end{aligned} \quad (25)$$

Then, we have

$$(p_0 + \tau_0) \% N_0 = p_0 + \tau_0 - \epsilon N_0, \quad (26)$$

$$\lfloor (p_0 + \tau_0) / N_0 \rfloor = \epsilon. \quad (27)$$

From (18), the periodic correlation function between $M_{n_0}^{(k_0)}$ and $M_{n_1}^{(k_1)}$ can be represented as

$$\begin{aligned} R_{M_{n_0}^{(k_0)}, M_{n_1}^{(k_1)}}(\tau) &= \sum_{p=0}^{PN_0-1} m_p^{(k_0, n_0)} \cdot m_{(p+\tau)\%PN_0}^{(k_1, n_1)*} \\ &= \sum_{p_0=0}^{N_0-1} o_{p_0}^{(n_0)} \cdot o_{p_0+\tau_0-\epsilon N_0}^{(n_1)*} \\ &\quad \cdot \sum_{p_1=0}^{P-1} s((k_0+p_0N_1)N_2+p_1)\%P \\ &\quad \cdot s^*((k_1+(p_0+\tau_0-\epsilon N_0)N_1)N_2+p_1+\tau_1+\epsilon)\%P \\ &= \sum_{p_0=0}^{N_0-1} o_{p_0}^{(n_0)} \cdot o_{p_0+\tau_0-\epsilon N_0}^{(n_1)*} \\ &\quad \cdot R_{S,S}((k_1-k_0)N_2 + (\tau_0 - \epsilon N_0)N_1N_2 \\ &\quad + \tau_1 + \epsilon). \end{aligned} \quad (28)$$

For the sake of simplicity, we introduce the following notation:

$$\rho = ((k_1 - k_0)N_2 + (\tau_0 - \epsilon N_0)N_1N_2 + \tau_1 + \epsilon) \% P, \quad (29)$$

$$\rho_0 = \tau_0N_1N_2 + \tau_1, \quad (30)$$

$$\rho_1 = \epsilon(P-1) + (k_0 - k_1)N_2, \quad (31)$$

$$\rho = (\rho_0 - \rho_1) \% P. \quad (32)$$

Note that $N_0N_1N_2 = P$. Since S is a perfect sequence, $R_{S,S}(\rho)$ becomes E_S when $\rho = 0$, and it becomes 0 when $\rho \neq 0$. Now, we consider the following two cases.

[A] Suppose that $k_0 = k_1$; then ρ_1 is represented as

$$\rho_1 = \epsilon(P-1). \quad (33)$$

In addition, suppose that $0 \leq \tau \leq P-1$; then from (20), (21), and (22),

$$0 \leq \tau_0 \leq N_0 - 1, \quad (34)$$

$$0 \leq \tau_1 \leq N_1N_2 - 1. \quad (35)$$

Therefore, the range of ρ_0 becomes

$$0 \leq \rho_0 \leq P-1. \quad (36)$$

Suppose that $\epsilon = 0$; then $\rho_0 = \rho_1$ holds if and only if $\rho_0 = 0$, i.e.,

$$\tau_0 = \tau_1 = 0. \quad (37)$$

This means that $\tau = 0$. On the other hand, suppose that $\epsilon = 1$; then $\rho_0 = \rho_1$ holds if and only if $\rho_0 = P-1$, i.e.,

$$\tau_0 = N_0 - 1, \quad (38)$$

$$\tau_1 = N_1N_2 - 1. \quad (39)$$

This means that $\tau = P-1$. Therefore, $R_{S,S}(\rho) = 0$ is satisfied, at least in the range of $1 \leq \tau \leq P-2$. Based on (28), this means that

$$\begin{aligned} \forall k_0, \forall n_0, \forall n_1, 1 \leq \tau \leq P-2, \\ R_{M_{n_0}^{(k_0)}, M_{n_1}^{(k_0)}}(\tau) &= 0. \end{aligned} \quad (40)$$

The periodic correlation function has the following symmetric property [28]:

$$R_{M_{n_0}^{(k_0)}, M_{n_1}^{(k_1)}}(-\tau) = R_{M_{n_1}^{(k_1)}, M_{n_0}^{(k_0)}}^*(\tau). \quad (41)$$

Exchanging n_0 and n_1 in (40) and applying (41), we have

$$\begin{aligned} \forall k_0, \forall n_0, \forall n_1, -P+2 \leq \tau \leq -1, \\ R_{M_{n_0}^{(k_0)}, M_{n_1}^{(k_0)}}(\tau) = 0. \end{aligned} \quad (42)$$

When $\tau = 0$, we have $\tau_0 = \tau_1 = \epsilon = 0$ from (20) and (24). Therefore, (28) becomes

$$\begin{aligned} R_{M_{n_0}^{(k_0)}, M_{n_1}^{(k_0)}}(\tau) &= \sum_{p_0=0}^{N_0-1} o_{p_0}^{(n_0)} o_{p_0}^{(n_1)*} R_{S,S}(0) \\ &= E_S \sum_{p_0=0}^{N_0-1} o_{p_0}^{(n_0)} o_{p_0}^{(n_1)*} \end{aligned} \quad (43)$$

Since O is a set of orthogonal codes, (43) means that

$$\begin{aligned} \forall k_0, \forall n_0 \neq n_1, \\ R_{M_{n_0}^{(k_0)}, M_{n_1}^{(k_0)}}(0) = 0. \end{aligned} \quad (44)$$

From (40), (42), and (44), each M_k is a quasi-optimal ZCZ sequence set that is represented as $Z_{cz}(PN_0, N_0, P-2)$.

[B] Suppose that $k_0 \neq k_1$. In addition, suppose that $0 \leq \tau \leq P/N_1 - 1$; then from (20), (21), and (22),

$$0 \leq \tau_0 \leq N_0 - 1, \quad (45)$$

$$0 \leq \tau_1 \leq N_2 - 1. \quad (46)$$

Note that $P/N_1 = N_0N_2$. Therefore, the range of ρ_0 becomes

$$\tau_0N_1N_2 \leq \rho_0 \leq \tau_0N_1N_2 + N_2 - 1. \quad (47)$$

Note that the minimum value of ρ_0 is 0 and the maximum value of ρ_0 is $P - N_1N_2 + N_2 - 1 < P$. Since $k_0 \neq k_1$, the range of $k_0 - k_1$ becomes

$$-N_1 + 1 \leq k_0 - k_1 \leq -1, 1 \leq k_0 - k_1 \leq N_1 - 1. \quad (48)$$

Suppose that $\epsilon = 0$; then ρ_1 is represented as

$$\rho_1 = (k_0 - k_1)N_2. \quad (49)$$

The range of ρ_1 becomes

$$-N_1N_2 + N_2 \leq \rho_1 \leq -N_2, N_2 \leq \rho_1 \leq N_1N_2 - N_2. \quad (50)$$

Since the minimum value of $\rho_0 - \rho_1$ is $-N_1N_2 + N_2 > -P$ and the maximum value of $\rho_0 - \rho_1$ is $P - 1$, $(\rho_0 - \rho_1) \% P = 0$ means $\rho_0 = \rho_1$. Since ρ_1 is a multiple of N_2 , the following condition is necessary in order to satisfy $\rho_0 = \rho_1$:

$$\tau_0N_1N_2 = (k_0 - k_1)N_2. \quad (51)$$

However, $k_0 - k_1$ is not a multiple of N_1 from (48). Therefore, $\rho_0 \neq \rho_1$, i.e., $\rho \neq 0$ if $\epsilon = 0$. On the other hand, suppose that $\epsilon = 1$; then ρ_1 is represented as

$$\rho_1 = P - 1 + (k_0 - k_1)N_2. \quad (52)$$

The range of ρ_1 becomes

$$P - 1 - N_1N_2 + N_2 \leq \rho_1 \leq P - 1 - N_2, \quad (53)$$

$$P - 1 + N_2 \leq \rho_1 \leq P - 1 + N_1N_2 - N_2. \quad (54)$$

From (25), if $\epsilon = 1$, $\tau_0 \neq 0$ is satisfied. Therefore, the minimum value of ρ_0 is N_1N_2 . Since the minimum value of $\rho_0 - \rho_1$ is $-P + 1 + N_2 > -P$ and the maximum value of $\rho_0 - \rho_1$ is 0, $(\rho_0 - \rho_1) \% P = 0$ means $\rho_0 = \rho_1$. Since ρ_1 is one less than a multiple of N_2 , the following condition is necessary in order to satisfy $\rho_0 = \rho_1$:

$$\tau_0N_1N_2 + N_2 - 1 = N_0N_1N_2 - 1 + (k_0 - k_1)N_2. \quad (55)$$

Note that $\tau_1 = N_2 - 1$ in this case. (55) can be represented as

$$(\tau_0 - N_0)N_1 = k_0 - k_1 - 1. \quad (56)$$

(56) holds if and only if $k_0 - k_1 = -N_1 + 1$ and $\tau_0 = N_0 - 1$. Since $\tau_0 = N_0 - 1$ and $\tau_1 = N_2 - 1$, we have $\tau = P/N_1 - 1$. Therefore, $R_{S,S}(\rho) = 0$ is satisfied, at least in the range of $0 \leq \tau \leq P/N_1 - 2$. Based on (28), this means that

$$\begin{aligned} \forall k_0 \neq k_1, \forall n_0, \forall n_1, 0 \leq \tau \leq P/N_1 - 2, \\ R_{M_{n_0}^{(k_0)}, M_{n_1}^{(k_1)}}(\tau) = 0. \end{aligned} \quad (57)$$

Similar to [A], we have

$$\begin{aligned} \forall k_0 \neq k_1, \forall n_0, \forall n_1, -P/N_1 + 2 \leq \tau \leq 0, \\ R_{M_{n_0}^{(k_0)}, M_{n_1}^{(k_1)}}(\tau) = 0. \end{aligned} \quad (58)$$

From (57) and (58), when we regard M as a single sequence set, M is also a quasi-optimal ZCZ sequence set that is represented as $Z_{cz}(PN_0, N_0N_1, P/N_1 - 2)$.

Thus, the proposed method has been proven.

V. EXAMPLE

In this section, we give a simple example.

Suppose that $P = 16$ and

$$S = (0000012302020321),$$

where 0, 1, 2, and 3 represent $+1$, $+\sqrt{-1}$, -1 , and $-\sqrt{-1}$, respectively. Then, S is a quadriphase perfect sequence of period 16. In addition, suppose that $N_0 = 4$ and

$$O_0 = (0000), O_1 = (0123), O_2 = (0202), O_3 = (0321).$$

Then, $O = \{O_n \mid 0 \leq n \leq 3\}$ is a set of four quadriphase orthogonal codes of length 4. Let $N_1 = 4$ and $N_2 = 1$; then a quadriphase MO-ZCZ sequence set with four ZCZ sequence sets containing four sequences of period 64 is obtained from (18). Each sequence of this MO-ZCZ sequence set is given

below:

$$\begin{aligned}
M_0^{(0)} &= (00000123020203210000123020203210 \\
&\quad 00002301020221030000301220201032), \\
M_1^{(0)} &= (01230202032100000123131321033333 \\
&\quad 01232020032122220123313121031111), \\
M_2^{(0)} &= (02020321000001230202103222223012 \\
&\quad 02022103000023010202321022221230), \\
M_3^{(0)} &= (03210000012302020321111123013131 \\
&\quad 03212222012320200321333323011313), \\
M_0^{(1)} &= (01230202032100001230202032100000 \\
&\quad 23010202210300003012202010320000), \\
M_1^{(1)} &= (02020321000001231313210333330123 \\
&\quad 20200321222201233131210311110123), \\
M_2^{(1)} &= (03210000012302021032222230120202 \\
&\quad 21030000230102023210222212300202), \\
M_3^{(1)} &= (00000123020203211111230131310321 \\
&\quad 22220123202003213333230113130321), \\
M_0^{(2)} &= (02020321000012302020321000002301 \\
&\quad 02022103000030122020103200000123), \\
M_1^{(2)} &= (03210000012313132103333301232020 \\
&\quad 03212222012331312103111101230202), \\
M_2^{(2)} &= (00000123020210322222301202022103 \\
&\quad 00002301020232102222123002020321), \\
M_3^{(2)} &= (01230202032111112301313103212222 \\
&\quad 01232020032133332301131303210000), \\
M_0^{(3)} &= (03210000123020203210000023010202 \\
&\quad 21030000301220201032000001230202), \\
M_1^{(3)} &= (00000123131321033333012320200321 \\
&\quad 22220123313121031111012302020321), \\
M_2^{(3)} &= (01230202103222223012020221030000 \\
&\quad 23010202321022221230020203210000), \\
M_3^{(3)} &= (02020321111123013131032122220123 \\
&\quad 20200321333323011313032100000123).
\end{aligned}$$

Each $M_k = \{M_n^{(k)} \mid 0 \leq n \leq 3\}$ is a quasi-optimal ZCZ sequence set that is represented as $Z_{cz}(64, 4, 14)$. In addition, $M = \{M_k \mid 0 \leq k \leq 3\}$ is a single quasi-optimal ZCZ sequence set that is represented as $Z_{cz}(64, 16, 2)$. It is clear that M is an MO-ZCZ sequence set.

VI. CONCLUSIONS

We have proposed a new method for constructing MO-ZCZ sequence sets using perfect sequences, orthogonal codes, and the interleaving technique. Each ZCZ sequence set of the proposed MO-ZCZ sequence set becomes a quasi-optimal ZCZ sequence set. In addition, the proposed MO-ZCZ sequence set can be regarded as a single quasi-optimal ZCZ sequence

set. In this sense, the proposed MO-ZCZ sequence set can be regarded as a generalized version of conventional MO-ZCZ sequence sets. The proposed MO-ZCZ sequence sets are expected to be useful for designing spreading sequences for AS-CDMA systems.

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