RLS-Based Frequency Synchronization and Channel Estimation in OFDMA Systems

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***Abstract*—In this paper, we investigate the problem of carrier frequency-offset (CFO) synchronization and channel estimation in orthogonal frequency-division multiple access (OFDMA) systems operating over unknown frequency-selective fading channels. We first devise a novel iterative method for joint CFO and channel estimation, based on the time-domain training blocks. The proposed joint estimator consists of two recursive least-square (RLS) algorithms. We then propose a more precise pilot-aided RLS algorithm to estimate the residual frequency synchronization errors or track small CFO changes for each user. With this, the accuracy of channel estimation is also enhanced. The analysis and simulation results show that, the proposed estimation and tracking scheme which is fully compatible with the existing standards is able to attain fast convergence, high stability, and ideal performances in all ranges of signal-to-noise ratio (SNR). Moreover, it can work well for wide tracking range up to ±0.5 of the subcarrier spacing.**

***Keywords*— Orthogonal frequency-division multiple access (OFDMA), carrier frequency offset (CFO), channel impulse response (CIR), recursive least square (RLS) algorithm.**

I. INTRODUCTION

Next-generation wireless communication systems are required to support high-data-rate applications. A novel technique that has attracted a lot of attention is orthogonal frequency division multiplexing (OFDM) [1]-[2], and OFDM introduces overlapping but orthogonal narrowband sub-channels to convert a frequency selective fading channel into a non-frequency selective one. Moreover, OFDM avoids inter-symbol interference (ISI) by means of cyclic prefix (CP) [3].

Coherent detection which constitutes the best demodulation principle implies an accurate channel impulse response (CIR) estimation algorithm at the receiver. In addition, OFDMA exhibits pronounced sensitivity to carrier frequency offset (CFO) between each user and receiver on uplink, which introduces inter-carrier interference (ICI). Hence, the combination of CFO and CIR estimation leads to particularly complex problems in OFDMA systems due to the number of unknowns.

This issue have been received a lot of attentions in OFDM context and several solutions are now available (see, e.g., [4]-[6] and references therein). However, all of them are designed for OFDM systems rather than OFDMA, where several users simultaneously transmit their own data by set of orthogonal subcarriers in uplink. Only a few methods for frequency synchronization and channel estimation for OFDMA system have been studied [7]-[11]. But most of them suffer from the high computational complexity (see e.g. [9] that needs to matrix inversion) or low CFO tracking range that is necessary condition for approximation that have been used in system model [10].

This paper is our effort to establish a new effective scheme for CFO synchronization and CIR estimation in OFDMA systems. We first propose a novel iterative joint CFO and CIR estimator, assuming time-domain training blocks are available. The proposed joint estimator consists of two RLS algorithms which iterate their estimated CFO and CIR values for each user at each iteration. Then, using the estimated CFO (for data block compensation) and CIR (in tracking system model) to further improve the CFO estimation accuracy, we derive a more precise pilot-aided RLS algorithm to estimate the residual errors or track small changes. With this, the accuracy of channel estimation is also enhanced. It is worthy to mention that the proposed CFO synchronization and CIR estimation scheme is fully compatible with the existing OFDM standards (like IEEE 802.11). Moreover, the proposed scheme can work well for initial frequency offset up to ±0.5 of the subcarrier spacing. Then, it can be used to cover a wide tracking range.

The rest of the paper is organized as follows. In the next Section, we describe the OFDMA signal model. Section III deals with the time-domain joint estimation of frequency-offset and channel coefficients. The pilot-aided CFO tracking algorithm is derived in Section IV. Simulation results are discussed in Section V and finally, the main results are summarized in Section VI.

The notation used in this paper is as follows. Bold letters denote matrix (uppercase) or vector (lowercase), and $(.)^T, (.)^*, Re(.)$ denote the transpose, the conjugate and real part of complex number operations.

II. SYSTEM MODEL

We consider an OFDMA communication system with N sub carriers, M user, signaling through an L-tap frequency-selective fading channel. Basically, the *m-th* user use $(k2_m - k1_m + 1)$ of subcarrier among of total N subcarrier ($k1_m$ and $k2_m$ are the first and the last subcarrier of dedicated subcarrier block for *m-th* user) to transmit data stream $\{X_{u,m}(k)\}$. Let us define the *u-th* OFDMA block to be transmitted from *m-th* user as

$$\boldsymbol{X}_{u,m} = [X_{u,m}(k1_m), \dots, X_{u,m}(k2_m)]^T_{(k2_m - k1_m + 1)\times 1} \quad (1)$$

where $X_{u,m}(k)$ denotes the *u-th* transmitted symbol from the *m-th* user at the *k-th* subcarrier. In addition to data symbols, each block is considered also includes K_P inserted pilots to help update/track channel estimation and synchronization for each user. Then, the corresponding time-domain symbol for *m-th* user is given by

$$x_{u,m}(n) = \frac{1}{\sqrt{N}} \sum_{k=k1_m}^{k2_m} X_{u,m}(k) e^{j\frac{2\pi k}{N}n}, n = 1, \dots, N \quad (2)$$

After cyclic prefix addition and pulse shaping, the time domain signal is modulated using carriers with local oscillator frequency f_c through *L-th* order frequency-selective fading channel. Like practical applications, data transmission is carried out in a burst manner. To facilitate quick synchronization, each burst is preceded with $P_{Short}+P_{Long}$ known training blocks (the preamble segment). The preamble is carefully designed to provide enough information for good burst detection, frequency offset estimation, symbol timing and channel estimation[1].

It is assumed that the fading process remains static during each transmitted burst but it varies from one burst to another, and the fading processes associated with different transmit–receive antenna pairs for each user are uncorrelated. The *L*-th order frequency-selective fading channel between *m-th* user and receiver is denoted by

$$\boldsymbol{h}_m = [h_m(0), \dots, h_m(L-1)]^T_{L\times 1}, \;\; 1 \le m \le M \quad (3)$$

At receiver, a superposition of faded signals from all users plus noise is received. Received signals are down converted to baseband with the local oscillators centered at $\hat{f}_c$. After S/P conversion and CP removal, the *n-th* received sample of the *u-th* OFDM block at the receive antenna is determined by

$$r_u(n) = \sum_{m=1}^{M} e^{j\frac{2\pi\varepsilon_m}{K}(n+N_u)} \sum_{l=0}^{L-1} h_m(l) x_{u,m}(n-l) + w_u(n) \quad (4)$$

where ε_m is the CFO normalized to the subcarrier spacing for *m-th* user, $n=1,\dots,N$, $N_u=N_{pre}+N_g+(u-1)(N+N_g)$, ($N_{pre}$ denotes the preamble length) and $w_m(n)$ is a zero-mean, additive white, complex Gaussian distributed noise sample with variance σ_w^2. The presence of CFO entails loss of orthogonality among subcarriers, which results in ICI and hence bit-error rate (BER) degradation. The goal is thus estimating ε_m. The estimation of CFO for all *m* users will be described in next sections.

[1] Assuming ideal synchronization in time, the frame detection and symbol timing will not be addressed in this paper.

Denoting $H_m(k) = \sum_{l=0}^{L-1} h_m(l) e^{-j\frac{2\pi kl}{N}}$, the corresponding frequency-domain sample for *m-th* user can be expressed as

$$Y_{u,m}(k) = \frac{1}{N} \sum_{n=0}^{N-1} \left\{ \sum_{m=1}^{M} \left[e^{j\frac{2\pi\varepsilon_m}{N}(n+N_u)} \sum_{i=k1_m}^{k2_m} H_m(i) X_{u,m}(i) e^{j\frac{2\pi(i-k)}{N}n} \right] \right\} + W_u(k) \quad (5)$$

where $W_u(k)$ represents the frequency-domain zero-mean, additive white, complex Gaussian distributed noise sample with variance σ_W^2.

III. Joint CFO and CIR Estimation

The proposed estimator consists of two RLS algorithms. The first one (CIR estimator) takes as input the time-domain received training symbols from all *M* users, (which is fed back by the second RLS algorithm at the previous iteration[2]) and computes as output the $\widehat{\boldsymbol{h}}_m^{(i)}$, (which is sent to the second RLS algorithm). The second RLS algorithm (CFO estimator) takes as input the time-domain received training symbols and $\widehat{\boldsymbol{h}}_m^{(i)}$. It computes the all CFOs $\hat{\varepsilon}_m^{(i)}$ for M users and then feeds it back to the CIR estimator and thus completes one iteration. At the end of the last iteration, the final CFO and CIR estimate values, $\hat{\varepsilon}_m$ and $\widehat{\boldsymbol{h}}_m$, are output by the joint estimator. In the following, we describe the performed calculations in each iteration of the joint estimator.

For both estimators, we used the received signal to form the $r_m(n)$ as time domain part of *m-th* user. On the other hand, we using the property of OFDMA transmission signal to extract each user transmitted signal part as

$$r_{u,m}(n) = \sum_{n'=0}^{N-1} r_u(n') g_m(n'-n) \quad (6)$$

where,

$$g_m(n'-n) = \frac{1}{N} \sum_{k=k1_m}^{k2_m} e^{j\frac{2\pi k(n'-n)}{N}}, -N+1 \le (n'-n) \le N-1$$

For compensation of this filtering (the effect of signal transmission to frequency domain, carrier separation and get it back to time domain using $g_m(n'-n)$ function), we express the time domain system model for *n-th* sample belongs to *m-th* user as

$$f_m\left(\hat{\varepsilon}_m^{(i)}, \widehat{\boldsymbol{h}}_m^{(i)}, \boldsymbol{x}_{m,i}^{g}\right) = \frac{1}{N} \sum_{l=0}^{L-1} h_m^{(i)}(l) x_m^g(i-l) \;,\;\; L \le i \le N_{pre} \quad (7)$$

where $x_m^g(i-l)$ expressed in equation (8) as

[2] We set the initial estimates $\hat{\varepsilon}_m^{(0)} = 0$ and $\widehat{\boldsymbol{h}}_m^{(0)} = 0$.

$$x_{m,i}^{g}(n-l)=\sum_{n'=0}^{N-1}x_m(n'-l)g_m(n'-n)e^{j\frac{2\pi\varepsilon_{m,pre}^{(i)}(n'+N_u)}{N}} \quad (8)$$

Based on the signal model in (3), the least-square (LS) estimate for $\boldsymbol{h}_m$ is obtained by minimizing the following cost function:

$$\xi(\widehat{\boldsymbol{h}}_m^{(i)})=\sum_{p=1}^{i}\lambda_1^{i-p}|e_m^h|^2 \quad (9)$$

where $\widehat{\boldsymbol{h}}_m^{(i)}$ and λ_1 denote the estimate of $\boldsymbol{h}_m$ at the *i-th* iteration of joint estimator and the forgetting factor, respectively (see [12] for more about RLS algorithm). From (3), we obtain:

$$e_m^h=r_m(n)-f_m(\hat{\varepsilon}_m^{(i-1)},\widehat{\boldsymbol{h}}_m^{(i-1)},\boldsymbol{x}_{m,i}^g) \quad (10)$$

The final expression for $\widehat{\boldsymbol{h}}_m^{(i)}$ is then obtained as

$$\widehat{\boldsymbol{h}}_m^{(i)}=\widehat{\boldsymbol{h}}_m^{(i-1)}+\boldsymbol{k}_m\, e_m^h \quad (11)$$

where $\boldsymbol{k}_m$, is defined by

$$\boldsymbol{k}_m=\frac{\boldsymbol{P}_{m,i-1}\,(\boldsymbol{x}_{m,i}^g)^*}{\lambda_m+\boldsymbol{x}_{m,i}^g\boldsymbol{P}_{m,i-1}\,(\boldsymbol{x}_{m,i}^g)^*} \quad (12)$$

for $\boldsymbol{x}_{m,i}^g=[x_{m,i}^g(i+L)\;x_{m,i}^g(i+L-1)\,,\dots,x_{m,i}^g(i+1)]$, and

$$\boldsymbol{P}_{m,i}=\frac{\boldsymbol{P}_{m,i-1}-\boldsymbol{k}_m\boldsymbol{x}_{m,i}^g\boldsymbol{P}_{m,i-1}}{\lambda_m} \quad (13)$$

We will shortly explain how $\widehat{\boldsymbol{h}}_m^{(i)}$ is then used for finding the $\hat{\varepsilon}_m^{(i)}$ by second RLS approach. To exploit the RLS approach for finding $\hat{\varepsilon}_m^{(i)}$, the following weighted squared error sum should be minimized,

$$\xi(\hat{\varepsilon}_m^{(i)})=\sum_{p=1}^{i}\lambda_2^{i-p}|e_m^{\varepsilon}|^2 \quad (14)$$

where, λ_2 represents the forgetting factor of the second RLS algorithm. From (3), we obtain

$$e_m^{\varepsilon}=r_m(i+L)-f_m(\hat{\varepsilon}_m^{(i-1)},\widehat{\boldsymbol{h}}_m^{(i)},\boldsymbol{x}_{m,i}^g) \quad (15)$$

In order to make use a linear RLS, a first-order Taylor series approximation is applied to the non-linear estimation error. Therefore [13],

$$e_m^{\varepsilon}\approx r_m(i+L)-\left\{f_m(\hat{\varepsilon}_m^{(i)},\boldsymbol{x}_{m,i}^g)+\frac{\partial f_m(\hat{\varepsilon}_m^{(i)},\boldsymbol{x}_{m,i}^g)}{\partial\hat{\varepsilon}_m^{(i)}}(\hat{\varepsilon}_m^{(i)}-\hat{\varepsilon}_m^{(i-1)})\right\} \quad (16)$$

where we obtain

$$\frac{\partial f_m}{\partial\hat{\varepsilon}_m}=\frac{1}{N}\sum_{l=0}^{L-1}\left\{h_m(l)\sum_{n'=0}^{N-1}\left[j\frac{2\pi(n'+N_u)}{N}x_m(n'-l)\; g_m(n'-n)\,e^{j\frac{2\pi\varepsilon_{m,pre}^{i}(n'+N_u)}{N}}\right]\right\} \quad (17)$$

Subsequently, the final expression for $\hat{\varepsilon}_m^{(i)}$ is obtained as

$$\hat{\varepsilon}_m^{(i)}=\hat{\varepsilon}_m^{(i)}+Re(e_m^{\varepsilon}k_m) \quad (18)$$

where e_m^{ε} and k_m, respectively, are defined as

$$e_m^{\varepsilon}=\frac{r_m(n)-\boldsymbol{h}_{pre}^{i-1^T}\boldsymbol{x}_{m,i}^g}{1+\frac{\partial f_m}{\partial\hat{\varepsilon}_m}k_m} \quad (19)$$

$$k_m=\frac{p_{m,i-1}\left(\frac{\partial f_m}{\partial\hat{\varepsilon}_m}\right)^*}{\lambda_m+\frac{\partial f_m}{\partial\varepsilon_m}p_{m,i-1}\left(\frac{\partial f_m}{\partial\hat{\varepsilon}_m}\right)^*} \quad (20)$$

and,

$$p_{m,i}=\frac{p_{m,i-1}-k_m\frac{\partial f_m}{\partial\hat{\varepsilon}_m}p_{m,i-1}}{\lambda_m} \quad (21)$$

Based on (9) and (14), we form a novel method for joint estimation of CFO and CIR. Proposed estimator iteratively provides the channel and frequency offset estimates. To regain the orthogonallity between the sub-carriers, the $\hat{\varepsilon}_m$ is removed from time-domain received samples in (4) before applying the discrete Fourier transform (DFT) operation, and signal separation for each user. However, the $\widehat{\boldsymbol{h}}_m$ will be used by tracking algorithm [See equations (22) and (23)].

IV. TRACKING USING PILOTS

The frequency offset requires constant tracking. This is aided by inserting known pilot symbols at fixed positions. In this section, to further improve the CFO estimation accuracy, we derive a more precise pilot-aided RLS algorithm to estimate the residual errors. With this, the accuracy of channel estimation is also enhanced.

Based on (5), the system model for *p-th* pilot of the *u-th* received OFDM block belongs to *m-th* user can be expressed as

$$f_m(k_p) = \frac{1}{N} X_m(k_p) \widehat{H}_m(k_p) \sum_{n=0}^{N-1} e^{j\frac{2\pi\check{\varepsilon}_m(n+N_u)}{N}} \tag{22}$$

where $\widehat{H}_m(k_p)$ could expressed as

$$\widehat{H}_m(k_p) = \sum_{l=0}^{L-1} \widehat{h}_m(l) e^{-j\frac{2\pi l}{N}k_p} \tag{23}$$

where $\check{\varepsilon}_m$ denotes the frequency offset estimation error for *m-th* user, $\varepsilon_m - \hat{\varepsilon}_m$. Using the same strategy as those given in the previous subsection, the final expression for the *i-th* estimate of $\check{\varepsilon}_m$ is obtained as[3]

$$\check{\varepsilon}_m^{(i)} = \check{\varepsilon}_m^{(i-1)} + Re(k_m e_m) \tag{24}$$

where e_m is defined by

$$e_m = \frac{R_m^c(k_p) - f_m(k_p)}{1 + \frac{\partial f_m}{\partial \varepsilon_m} k_m} \tag{25}$$

where we obtain

$$\frac{\partial f_m}{\partial \check{\varepsilon}_m} = \frac{1}{N} X_m(k_p) \widehat{H}_m(k_p) \sum_{n=0}^{N-1} j \frac{2\pi(n+N_u)}{N} e^{j\frac{2\pi\check{\varepsilon}_m^{(i-1)}(n+N_u)}{N}} \tag{26}$$

and $R_m^c(k_p)$, is compensated frequency domain received signal that could be expressed as

$$R_m^c(k) = \frac{1}{\sqrt{N}} \sum_{n=k1_m}^{k2_m} r(n) e^{-j\frac{2\pi\hat{\varepsilon}_m(n+N_u)}{N}} e^{-j\frac{2\pi nk}{N}} \tag{27}$$

for $k1_m \leq k \leq k2_m$. It is noted that the use of $\frac{1}{K}\sum_{n=1}^{K} e^{j\frac{2\pi\check{\varepsilon}^{(i)}}{K}n} \approx sinc(\check{\varepsilon}^{(i)}) e^{j\pi\check{\varepsilon}^{(i)}}$ results in an approximate expression of (22) that was used in [4]. Moreover,

$$k_m = \frac{p_{m,i-1}\left(\frac{\partial f_m}{\partial \check{\varepsilon}_m}\right)^*}{\lambda_m + \frac{\partial f_m}{\partial \check{\varepsilon}_m} p_{m,i-1}\left(\frac{\partial f_m}{\partial \check{\varepsilon}_m}\right)^*} \tag{28}$$

and,

$$p_{m,i} = \frac{p_{m,i-1} - k_m \frac{\partial f_m}{\partial \check{\varepsilon}_m} p_{m,i-1}}{\lambda_m} \tag{29}$$

[3] We set the initial estimate $\check{\varepsilon}_m^{(0)} = 0$.

As before mentioned, imperfect frequency synchronization degrades the accuracy of channel estimation, which in turn, affects the overall receiver performance. Consequently, after CFO tracking is performed, the channel estimation which is performed without considering $\check{\varepsilon}_m$ can be refined [as stated in equations (10)-(13)] for all M users. Depends on the performance we are looking for, the number of iterations for both CFO and CIR estimation will change.

V. Simulation Results

We provide computer simulation results to illustrate the performance of the proposed scheme. In the following simulations, we set the OFDM-related parameters based on the IEEE 802.11a un-coded systems [14]. The signal constellation of the quadrature phase shift keying (QPSK) is employed for the OFDMA blocks of 48/M data subcarriers, N=64 and 4 equally spaced pilot tones of the same power for M=2. For each user, a burst format of two random training blocks and 225 data OFDMA blocks are transmitted simultaneously. For each transmit–receive antenna pair, we considered a Rayleigh fading channel with *L*=7 taps and RMS delay spread of 50 (ns).

For reducing the computational complexity we can use the reduced form of separation filter (g_m). Figure 1 shows the real and imaginary parts of separation filter which we used for separation of first user transmitted signal in simulation result (where $k1_m = 1$ and $k2_m = 26$). As shown in Figure 1, the most points of g_m could be neglected. In Figure 2 the MSE (mean square error) performance of the tracking CFOs versus the number of iterations (the number of OFDMA symbols where each user use K_P/M pilot at each OFDMA symbol) is depicted (for λ = 0.99 and the regularization parameter γ = 10). In Figure 3, we depict the MSE and NMSE (normalized MSE) performance of the proposed pilot-aided tracking algorithm versus SNR based on equations (30) [15] and (31) [16] after 100 iterations. In Figure 3, the results are given for both complete and reduced (Method #2)

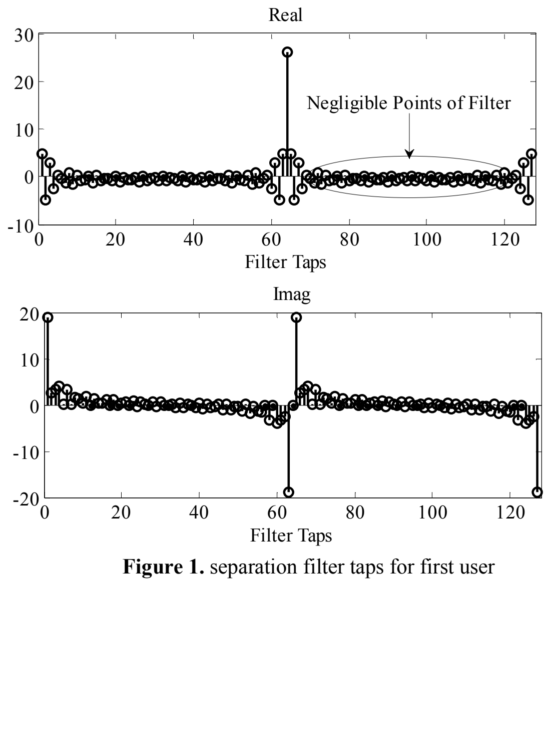

Figure 1. separation filter taps for first user

separation filter (g_m). The results also compared with ideal CRB (we assumed complete knowledge of CFO for CIR CRB calculations and complete knowledge of CIR for CFO CRB calculations) in practical range of SNR.

$$MSE_{CFO} = \frac{1}{M}\sum_{m=1}^{M}\left(\varepsilon_m - (\hat{\varepsilon}_m + \check{\varepsilon}_m)\right)^2 \quad (30)$$

$$NMSE_{CIR} = \frac{\sum_{m=1}^{M}\sum_{l=0}^{L-1}\left(h_m(l) - \hat{h}_m(l)\right)^2}{\sum_{m=1}^{M}\sum_{l=0}^{L-1}\left(h_m(l)\right)^2} \quad (31)$$

VI. Conclusions

In this paper, with the aid of training symbols in both time and frequency domains, we proposed a novel two-step scheme for frequency synchronization and channel estimation in OFDMA systems. The performance of the proposed estimation and tracking scheme is benchmarked with CRBs, and investigated by computer simulations. Simulation results show that the proposed CFO and CIR estimators achieve acceptable performance compared with the CRBs especially in low ranges of SNR with a wide tracking range of CFOs.

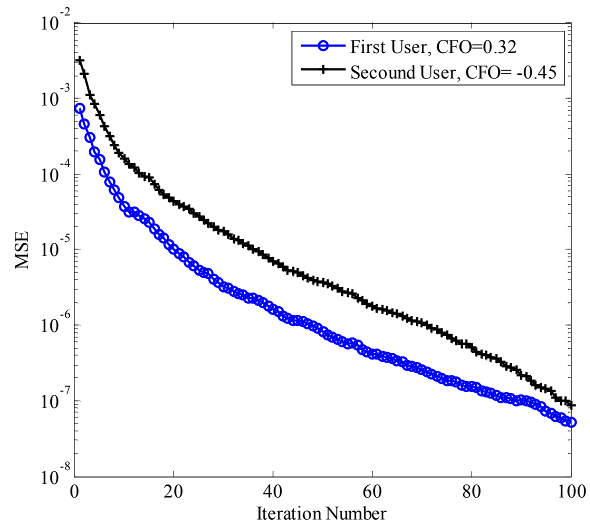

Figure 2. Learning curves for CFO tracking for M=2, SNR=10 dB and 7-tap frequency selective fading channel

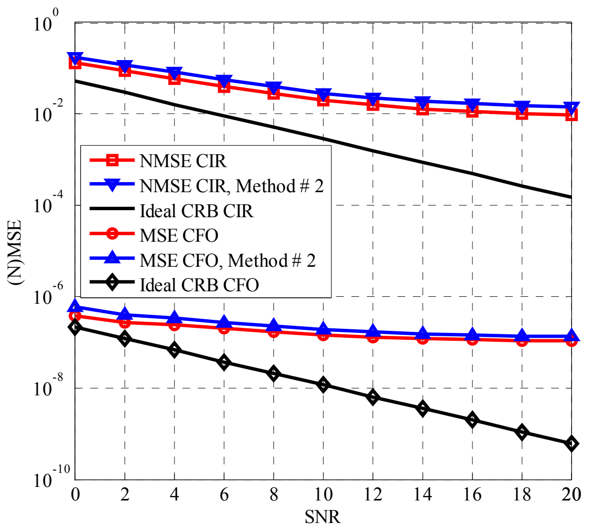

Figure 3. (N)MSE for CFO and CIR final estimation, CFO distributed in ± 0.5 at 7-tap fading channel for 2 user

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