A New Weighting Factor of PTS OFDM with Low Complexity Based on Split-Radix IFFT for PAPR Reduction

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Abstract— **The Partial Transmit Sequence (PTS) method with low computation complexity, called decomposition PTS sub-blocking was proposed which employs the radix-2 inverse fast Fourier transform (IFFT) for the signals at the middle stages of an N-point radix-2 IFFT and decimation in frequency (DIF) domain. This method (DIF-IFFT) can reduce the computation complexity relatively with keeping the better PAPR performance similar to other PTS techniques with using the same weighting factor. To improve computation complexity for the PTS method, the Split-Radix inverse fast Fourier transform (SRIFFT) which can reduces the number of complex computation was proposed. However, the PAPR reduction performance is the same as that for the radix-2 method. In this paper, we propose a new weighting factor technique in conjunction with DIF-PTS sub-blocking based on Split-Radix IFFT technique called Improve PTS (I-PTS) which can improve both the PAPR performance and computation complexity without any increasing of side information. This paper presents the various computer simulation results to verifier the effectiveness of proposed method.**

Index Terms— **DIF-IFFT, PAPR, SRIFFT, PTS.**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) method has been standardized as the European digital audio broadcasting (DAB) and the digital video broadcasting (DVB) systems. It has also been proposed for the third-generation mobile radio standard in Europe. Multiplexing a serial data symbol stream into a large number of orthogonal subcarriers makes the OFDM signals spectral bandwidth efficiently [1]. However, a major drawback of OFDM method is that the OFDM signal has higher peak to average power ratio (PAPR). The higher PAPR leads the fatal degradation of OFDM performance in the nonlinear power amplifier (HPA) [2].

Partial transmit sequence (PTS) method [3] is proposed as one of the distortion-less PAPR reduction methods. However, the computation complexity and the size of side information would increase as increasing the number of clusters and weighting factors. To reduce this computation complexity, DIF-PTS method was proposed [4] which employs the intermediate signals within the IFFT and used radix-2 decimation in the frequency domain (DIF) to obtain the PTS sub-blocks. Multiple IFFTs are then applied to the remaining stages. The PTS sub-blocking is performed in the middle stages of the N-point radix FFT DIF algorithm. The DIF-PTS method reduces the computational complexity relatively while it shows almost the same PAPR reduction performance as that of the conventional PTS OFDM scheme.

In this paper, we propose a new weighting factor technique for the PTS method in conjunction with DIF-PTS sub-blocking based on Split-Radix IFFT technique which can improve both the PAPR performance and computation complexity. The proposed method can achieve the better PAPR reduction performance than that for the DIF-PTS method without any increasing of the size of side information.

In the next section, the PAPR problem and conventional PTS are reviewed briefly. Section III presents the PTS-base Split-radix technique and Section IV presents the proposed method. Section V presents various computer simulation results to verify the effectiveness of the proposed method as comparing with the conventional PTS method. Some conclusions are given in Section VI.

II. PAPR PROBLEM AND CONVENTIONAL PTS METHOD

Let $\{X(k)\}_{k=0}^{N-1}$ denote the frequency-domain signal, where N is the number of FFT/IFFT points and k is the frequency index. The discrete time-domain OFDM signal is obtained by taking an N-point inverse discrete Fourier transform (IDFT) of $X(k)$ as given by the following equation.

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) T_N^{-nk}$$
$$k = 0,1,\ldots,N-1 \quad n = 0,1,\ldots,N-1 \tag{1}$$

where n is the discrete-time index, $T_N = e^{-j2\pi/N}$ (known as the twiddle factor), and $j^2 = -1$. The frequency-domain signal $X(k)$ would add constructively and generate a time domain signal with large peak amplitude. To evaluate the envelop variations of OFDM time domain signal, the ratio of peak to envelope power of the signal is usually used. The discrete time PAPR can be evaluated precisely by using a minimum of four times oversampling [5], which is given by,

$$PAPR = \frac{\max\limits_{0\le n\le N-1} |x(n)|^2}{E\left[|x(n)|^2\right]} \tag{2}$$

In the PAPR reduction method of using the partial transmit sequences (PTSs), the frequency-domain vector $X(k)$ is partitioned into P disjoint sub-blocks $X_p(k), p = 0,1\ldots,P-1$, so that $X(k) = \sum_{p=0}^{P-1} X_p(k)$. Let θ_p be the set of weighting factors with $\theta_1 = 0$ which are applied to the sub-blocks $X_p(k)$. The substitute frequency-domain signal are given by [6],

$$X^{'}(k) = \sum_{p=0}^{P-1} e^{j\theta_p} X_p(k) \cdot \quad (3)$$

Taking the IDFT of (3), and using the linearity property of the IDFT, the following equation can be obtained.

$$x^{'}(n) = IDFT(X^{'}(k)) = \sum_{p=0}^{P-1} e^{i\theta_p} IDFT(X_p(k))$$
$$= \sum_{p=0}^{P-1} e^{j\theta_p} x_p(n) \quad (4)$$

where $x_p(n) = IDFT(X_p(k))$ are the P time-domain PTSs. To determine the sequence $x^{'}(n)$ with the smallest PAPR, the following optimization criterion is employed.

$$\left[\theta_1^{'}, \theta_2^{'}, ..., \theta_{P-1}^{'}\right] = \underset{[\theta_1, \theta_2, ..., \theta_{P-1}]}{\arg\min} \left\{ \max_{0 \le n \le N-1} \left| x^{'}(n) \right| \right\} \cdot \quad (5)$$

In order to recover the data correctly at the receiver, the required side information is $(P-1)\log_2 W$ bits per OFDM symbol where W is the number of weighting factors. According to (4), P IDFTs are required to obtain $x^{'}(n)$ which can incur significant computational complexity.

III. PTS-Based Split-Radix Technique

A. Split-Radix FFT Algorithms

A Radix-2 algorithm diagram can be transformed quite straightforwardly into a Radix-4 algorithm diagram simply by changing the exponents of the twiddle factors. It is quite clear then that at each stage of the algorithm of Radix-4 is better for the odd terms of the DFT and Radix-2 for the even terms of the DFT. So, one might guess that restricting this transformation locally to the lower part of the diagram might improve the algorithm. It turns out that this is indeed the case. The Split-Radix algorithms is then based on the following decomposition [7]:

$$X_k = \sum_{n=0}^{N-1} x_n T_N^{nk}$$

if

$$\left(T_N = \cos\frac{2\pi}{N} - j\sin\frac{2\pi}{N} \right)$$

is the DFT to be computed, it is decomposed into,

$$X_{2k} = \sum_{n=0}^{N/2-1} (x_n + x_{n+(N/2)}) T_N^{2nk}$$

$$X_{4k+1} = \sum_{n=0}^{N/4-1} (x_n - x_{n+(N/2)}) - j(x_{n+(N/4)} - x_{n+3(N/4)}) T_N^{n} T_N^{4nk} \quad (6)$$

$$X_{4k+3} = \sum_{n=0}^{N/4-1} (x_n - x_{n+(N/2)}) + j(x_{n+(N/4)} - x_{n+3(N/4)}) T_N^{3n} T_N^{4nk}$$

The first stage of a Split-Radix decimation in the frequency decomposition then replaces a DFT of length N by one DFT of length $N/2$ and two DFT's of length $N/4$ at the cost of ($N/2$-4) general complex multiplications (3 real multiplications + 3 additions), and 2 multiplications by the eighth root of unity (2 real multiplications + 2 additions).

The length-N DFT is then obtained by successive use of such decompositions, up to the last stage where some usual radix-2 butterflies (without twiddle factors) are needed (see Fig.1 for a length 16-Split-Radix FFT).

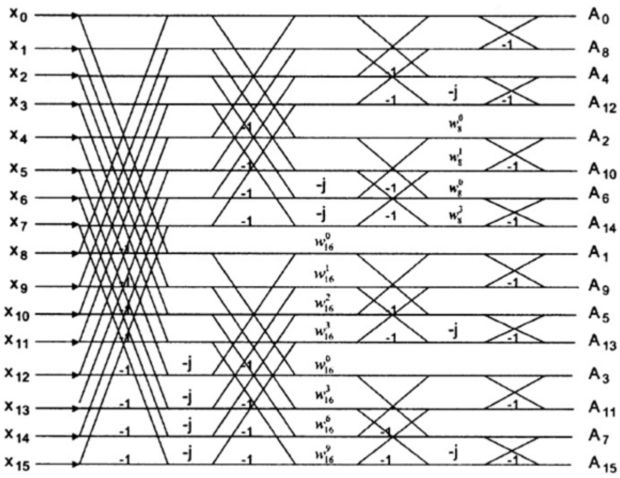

Fig. 1. Length 16 Split-Radix algorithms.

B. IFFT-Based PTS Technique

The DFT of an N-point sequence $X(k)$ can be directly computed by using equation (1). As the IDFT can be computed by taking the complex conjugate of the input and output sequences while using the same DFT parameters, we need only consider the DFT calculation. Thus, we only use the corresponding FFT computation in the following.

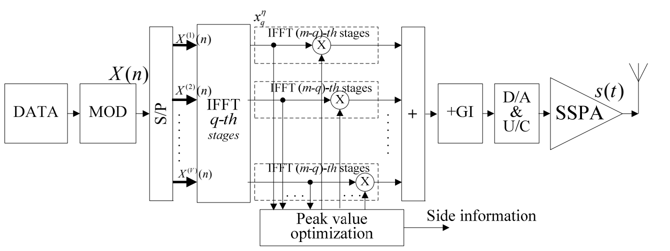

Fig. 2. Structure of OFDM transmitter with a low complexity PTS method.

An FFT algorithm recursively converts the DFT computation to $r \times N/r$- point DFTs recurring through $m = \log_r N$ stages. The value of r corresponds to a radix-r FFT algorithm. The DIF radix-r DFT of (1) is given by,

$$X(rk+k_0)=\sum_{n=0}^{N/r-1}\left(\left(\sum_{i=0}^{r-1}x\left(n+\frac{N}{r}i\right)T_r^{ik_0}\right)T_N^{nk_0}\right)T_{N/r}^{kn} \quad (7)$$

where $k_0, 0\le k_0\le r-1$, is the index of the butterfly outputs. As we consider the inputs to stage *q* for PTS sub-blocking, symbols and indices are represented with subscript $q: x_q$ and n_q for an input *x* and time index *n*, respectively, which X_q and k_q for an output X and frequency index *k*, respectively. Considering the form of (7), the butterfly outputs at stage *q* are given by,

$$X_q^\eta(rk_q+k_0)=\left(\sum_{i=0}^{r-1}x_q^\eta\left(n_q+\frac{N}{r^q}i\right)T_r^{ik_0}\right)T_{N/r^{q-1}}^{n_qk_0} \quad (8)$$

where k_q=0,1,…, $(N/r^q)-1$, n_q=0,1,…, $(N/r^q)-1$, and η, η=1,2,… r^{q-1}, denotes a particular N/r^{q-1}-point DFT at the stage *q*. Fig. 1 shows the recursive reduction of the η-th N/r^{q-1}-point DFT to N/r^q-point DFTs at stage *q*. It is assumed that the input sequence is in normal order, and the output is in digit-reversed order. Similarity, we can obtain the butterfly outputs at stage *q* for decimation in time (DIT) domain.

The inputs $X_q^\eta(n_q+(N/r^q)i)$ at stage *q* are used for cluster partitioning in the proposed PTS technique and the remaining *m-q* stages are used to compute the multiple transforms as shown in Fig. 2.

IV. PROPOSED METHOD

A. *Proposal of New Weighting Factor*

In the proposed method, the input data block is partitioned into the cluster as the same as conventional PTS method. The difference of proposed method as compared with the conventional method is that each cluster is partitioned by first and second parts as shown in Fig. 3. The first and second parts of cluster employ the different weighting factor although these two have the predetermined relationship [8]. The frequency domain signal for the proposed method can be given by,

$$x'(n)=\sum_{p=0}^{P-1}\left(e^{j\theta'_p}x'_p(n)+e^{j\theta''_p}x''_p(n)\right) \quad (9)$$

where $e^{j\theta'_p}$ and $e^{j\theta''_p}$ are weighting factors for the first and second parts at the *p-th* cluster, respectively. $x'_p(n)$ and $x''_p(n)$ are the data sub-carriers of first and second parts at the *p-th* cluster, respectively. The weighting factors of proposed method are given by the following equation.

$$\theta'_p=\chi\theta''_p$$
$$\theta''_p=\left\{\frac{2\pi i}{W}\middle| i=0,1,...,W-1\right\} \quad (10)$$

where θ'_p is the phase coefficient for the first part of clusters, and θ''_p is the phase coefficient for the second parts. However, this fact leads other advantage in the computational complexity for the proposed method as compared with the case of $\chi=0.5$ [9]. If α is 0, the phase value of second part of cluster can be obtained by $\phi_2^{(\nu)}=\alpha\phi_1^{(\nu)}=0$ and the weighting factor for the second part of cluster becomes $b_2^{(\nu)}=e^{j\phi_2^{(\nu)}}=1$. This means that the proposed method can use the original time domain signal without multiplying the weighting factor for the half part of subcarriers.

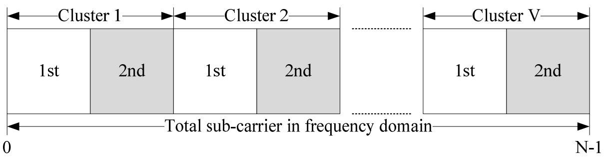

Fig. 3. Structure of OFDM symbol for the Improved PTS method.

From the above results, Fig. 4 shows the averaged PAPR performance for the proposed method when changing χ. The best PAPR performance can be achieved when χ is 0 and the proposed method with $\chi=0$ can reduce the computation complexity. From this fact, the proposed method shows better PAPR performance than conventional PTS and DIF-PTS method with keeping the same size of side information and lower computation complexity as the conventional PTS method.

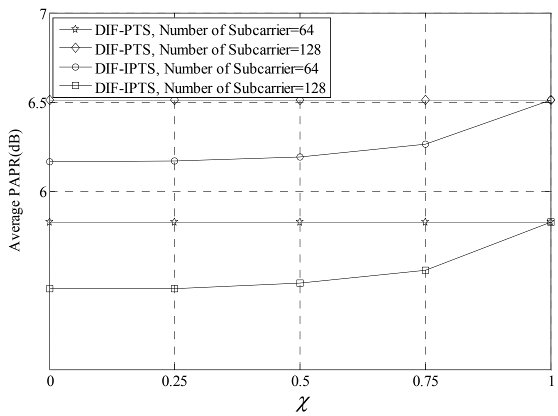

Fig. 4. Averaged PAPR Performance for the proposed DIF-IPTS method when changing χ.

B. *Computational Complexity Analyses*

We define the multiplicative complexity of the DIF IFFT algorithm as the number of complex multiplications by twiddle factors $T_{N/r^{q-1}}^{n_qk_0}$ and $T_r^{ik_0}$. The twiddle factors $T_r^{ik_0}$ are trivial (±1 and $\pm j$). Let M_m^c be the number of real multiplications needed to perform a 2^m-complex DFT with the Split-Radix algorithm. By using (6), we can obtain the following relationship.

$$M_m^c=M_{m-1}^c+2M_{m-2}^c+3\cdot2^{m-1}-8 \quad (11)$$

And, with the initial conditions M_1= 0, M_2= 0, we obtain,

$$M_m^c = 2^m(m-3)+4 \cdot \tag{12}$$

Disregarding for a while the number of additions needed to perform the complex multiplications, the remaining ones can easily be evaluated by $m \cdot 2^{m+1}$, since, at each of the m stage, anew point is generated by a complex addition. Then, since the number of real additions needed to compute a complex is equal to the number of multiplications, we have:

$$A_m^c = m \cdot 2^{m+1} + M_m^c \cdot \tag{13}$$

The Split-Radix algorithm has the lower number of both multiplications and additions than Radix-2 algorithm.

V. PERFORMANCE EVALUATION

This section presents the various computer simulation results to verify the performance of proposed method. The receiver is coherent detector. The transmitted signal is taken over sampling by a factor of 4 (*L*=4). The simulation parameters to be used in the following evaluations are listed in Table I.

TABLE I
SIMULATION PARAMETERS

Modulation	QPSK
Demodulation	Coherent
Allocated bandwidth	5MHz
Number of FFT points	256 and 512
Number of sub-carriers	64 and 128
Number of cluster (V)	4
Number of discrete phase (W)	4
Symbol duration	12.8us
Guard interval	1.28us

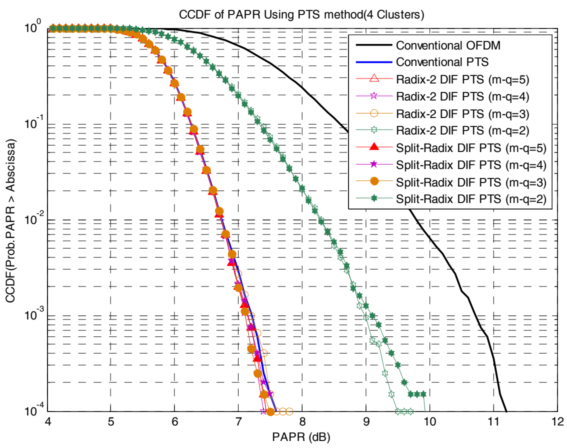

Fig. 5. Comparison of PAPR reduction performance for the conventional PTS, Radix-2 DIF PTS and Split-Radix DIF PTS methods.

Figure 5 shows the PAPR performance for the conventional OFDM, conventional PTS, DIF-PTS based on Radix-2 and Split-Radix, respectively when the modulation technique is QPSK. This figure shows the PAPR reduction performance of DIF-PTS method when the Radix-2 and Split-Radix are employed. The PAPR reduction performance both for the DIF-PTS base Radix-2 and Split-Radix can achieve same performance as that for the conventional PTS method when middle stages are $(m-q)_{Radix-2}=4$ and $(m-q)_{Split-Radix}=4$ $(m-q)_{Split-Radix}=4$ $(m-q)_{Split-Radix}=4$, respectively. However, the DIF-PTS can reduce much more computation complexity than conventional PTS method.

Table II shows the comparisons for the PAPR performance and computation complexity for the conventional OFDM, conventional PTS, DIF-PTS based on Radix-2 and Split-Radix, respectively. This table shows the comparison computation complexity which refers the conventional PTS. From the table, the DIF-PTS and DIF-IPTS based on radix-2 can reduce the computation complexity 68.76% at $(m-q)_{Radix-2}=2$ when comparing with conventional PTS. The DIF-IPTS based on Split-Radix shows the lower computation complexity which can reduce up to 81.08% at $(m-q)_{Split-Radix}=2$ when comparing with conventional method. From these results, it can be concluded that the proposed method can achieve the lower PAPR reduction performance and reduces the computation complexity as compared with the conventional PTS and DIF-PTS based on Radix-2.

TABLE II
COMPARISONS OF COMPUTATION COMPLEXITY FOR DIFFERENCE METHODS

	Computation multiplications Complexity (P=4 and N=256)				
	(m-q=6)	(m-q=5)	(m-q=4)	(m-q=3)	(m-q=2)
Conventional OFDM	NA	NA	NA	NA	NA
Conventional PTS	0%	0%	0%	0%	0%
DIF-PTS [4]	24.68%	36.77%	48.48%	59.40%	68.76%
Radix-2 DIF-IPTS	24.68%	36.77%	48.48%	59.40%	68.76%
Split-Radix DIF-IPTS	52.99%	59.04%	67.82%	74.64%	81.08%

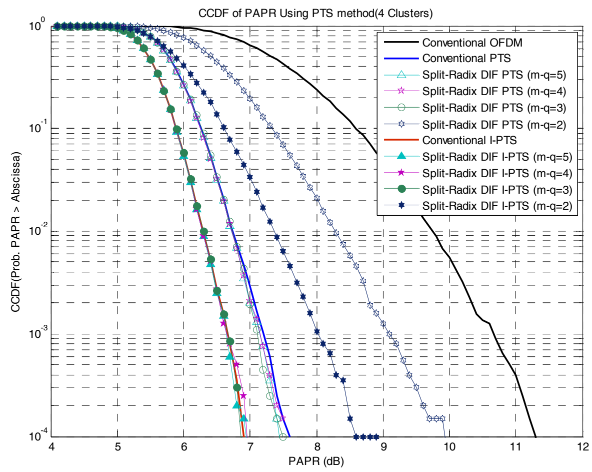

Fig. 6. Comparison of PAPR reduction performance between conventional PTS and Split-Radix DIF IPTS methods.

Figure 6 shows the PAPR performance for the conventional OFDM, conventional PTS, DIF-PTS and DIF-

IPTS based on Split-Radix, respectively when the modulation technique is QPSK, number of subcarriers is 64. From the figure, it can be observed that the PAPR reduction performance of DIF-IPTS method based on Split-Radix can achieve better PAPR reduction performance when comparing with DIF-PTS method based on Split-Radix. From the figure, it can be concluded that the proposed new weighting factor can achieve the lower computation complex than DIF-IPTS based Radix-2.

VI. CONCLUSIONS

In this paper, we proposed the new weighting factor technique for PTS method in conjunction with DIF-PTS method based on Split-Radix. The proposed new weighting factors for the 1st and 2nd parts have the predetermined relationship so as to keep the same size of side information. To reduce the computation complexity, we used the Split-Radix DIF-IFFT technique. From the computer simulation results, we confirmed that the proposed method shows the better PAPR performance and lower computation complexity with keeping the same size of side information as compared with the DIF-PTS method.

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