Low Complexity I/Q Imbalance and Channel Estimation Techniques for MIMO OFDM Systems

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Abstract—In this paper, the joint estimation techniques of in-phase and quadrature-phase (I/Q) imbalance and channel impulse response (CIR) are proposed for multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems over Rayleigh fading environments. A novel preamble-aided structure with orthogonal property is designed to cancel the image interference due to I/Q imbalance and channel effects. Simulation results confirm the proposed method with low computational complexity can accurately estimate the CIR and I/Q imbalance parameters and achieve better root mean square error (MSE) performance over RF impairment and multipath fading channel environments.

Keywords—SISO OFDM, MIMO OFDM, I/Q imbalance, channel estimation, preamble.

I. INTRODUCTION

Orthogonal frequency division multiplexing technique (OFDM) is applied widely to wireless communication systems. Besides, OFDM combines multiple-input and multiple-output (MIMO) can not only raise its data rate but also not occupy extra bandwidth. However, MIMO systems means that the architecture of receiver involves more complexity than single-input and single-output (SISO) systems. Figure 1 shows the black diagram of OFDM systems. At the receiver side, direct-conversion receiver and superheterodyne receiver are widely used. Although the performance of superheterodyne is better than direct-conversion, the consumption and complexity are also higher. Due to the requirement of low computational complexity and low power consumption, the direct-conversion receivers have drawn a lot of attentions and exhibited great advantages in the next-generation wireless communication systems. On the other hand, some gain and phase mismatches in direct conversion receiver can seriously degrade the system performance. One of these mismatches is so-called the in-phase and quadrature-phase (I/Q) imbalance which is induced by the mismatch of local oscillators. Especially, in the MIMO OFDM systems, I/Q imbalance leads to more obvious performance degradation. I/Q imbalance estimators and compensators have been studied by many researchers [1]-[8]. Among these works, the training sequences in [2]-[3] and optimal sequence in [4] are proposed to compensate the distortion of I/Q imbalance. They are still provide high computational complexity operation to estimate the parameters.

In this paper, we propose a preamble-aided method with low computational complexity advantage, which focuses on the estimation and compensation of the I/Q imbalance and multipath channel effects of MIMO OFDM system over the wireless Rayleigh fading channels [2], [5]. A novel preamble design, i.e., MIMO full-usage preamble sequence design, can jointly estimate the gain and phase parameters of I/Q imbalance and the channel impulse response (CIR). On one hand, by using the proposed preamble sequence, the parameters of I/Q imbalance and CIR can be accurately estimated and the image interference signal can be eliminated. On the other hand, it can equalize the received signal to detect the original transmitted data.

![Figure 1. The black diagram of OFDM](image)

II. SYSTEM AND SIGNAL MODEL

A. MIMO OFDM Signal Model

Figure 2 shows an MIMO OFDM systems where the number of transmit and receive antennas are $N_T$ and $N_R$, respectively. Assume there is no interference between each antenna and no inter symbol interference (ISI). We consider that channel effect which includes the wireless Rayleigh fading channel and additive white Gaussian noise (AWGN). The channel is given by
\[ H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N_t} \\ h_{21} & h_{22} & \cdots & h_{2N_t} \\ \vdots & \vdots & & \vdots \\ h_{N_{tx},1} & h_{N_{tx},2} & \cdots & h_{N_{tx},N_t} \end{bmatrix} \]  

(1)

Every element of \( H \) is individually independent complex Gaussian random variable with a zero-mean and unit variance. The signal model for MIMO systems is given by

\[ y_n = H_n x_n + n_n \]  

(2)

---

**Figure 2.** The channel of MIMO OFDM systems

Where \( y \in \mathbb{C}^{N_t} \), \( H \in \mathbb{C}^{N_{tx} \times N_t} \), \( n \in \mathbb{C}^{N_t} \) and \( m \) is \( m \)th symbol of OFDM signal. \( y = \left[ y_1, y_2, \ldots, y_{N_t} \right]^T \) is the received signal vector, \( x = \left[ x_1, x_2, \ldots, x_{N_t} \right]^T \) is the transmitted signal vector, \( n = \left[ n_1, n_2, \ldots, n_{N_t} \right]^T \) is AWGN vector. Note that, for MIMO OFDM systems, the channel of every subcarrier from different antennas can be regard as flat fading channel. The received signal of \( k \)th subcarrier can be written as (3). Where \( k = 0, \ldots, N_t - 1 \), and \( N_t \) is the length of subcarrier for OFDM.

\[ y_n(k) = H_n(k) x_n(k) + n_n(k) \]  

(3)

**B. The Received Signal Model with I/Q Imbalance**

The amplitude and phase mismatches of local oscillator are due to I/Q imbalance. This paper considers I/Q imbalance at receiver mainly. Figure 3 illustrates I/Q imbalance at receiver, where \( g \) and \( \psi \) denote the gain and phase mismatch respectively and \( \omega_c = 2\pi f_c t \), where \( f_c \) is the carrier frequency. In this paper, we will discuss effect of I/Q imbalance for SISO OFDM systems first, then extend it to MIMO OFDM systems.

**Figure 3.** The black diagram of I/Q imbalance

When I/Q imbalance exists at receiver side, the received signal for SISO OFDM systems can be modelled as [2]

\[ \hat{y}(t) = y(t) + jy_c(t) = R\{y(t)\} + jI\{ge^{-j\psi}y(t)\} = k_1 y(t) + k_2 y'(t) \]

(4)

where \( R\{\cdot\} \) and \( I\{\cdot\} \) denote real part and image part of received signal respectively, and \( k_1 = (1 + ge^{-j\psi}) \), \( k_2 = (1 - ge^{-j\psi}) \) are the parameters of I/Q imbalance. In the case of perfect matching between the I and Q branch, it means that \( g = 1 \) and \( \psi = 0^\circ \), and the parameters will be \( k_1 = 1 \), \( k_2 = 0 \). Assume there is perfectly synchronized for MIMO OFDM systems so that no interference between each signal. The received signal with I/Q imbalance can be extended from SISO OFDM systems to \( N_{tx} \times N_t \) MIMO OFDM systems, then transfer it from time domain to frequency domain, the received signal can be modelled as

\[ \hat{y}_m^{(N_t)}(k) = \sum_{n=1}^{N_t} (k^{(N_t)}(k)x_n^{(N_t)}(k) + k_2^{(N_t)}k_2^{(N_t)}(k)x_n^{(N_t)}(k) - k) x_n^{(N_t)}(k) \]

(5)

Where \( k_1^{(N_t)} \) and \( k_2^{(N_t)} \) are the parameters of I/Q imbalance of the \( N_{tx} \)th received antenna and \( x_n^{(N_t)}(k) \) represent the \( k \)th subcarrier of the \( m \)th OFDM symbol, where \( k \in \{-K, \ldots, -1, 0, \ldots, K\} \). We can observe the effect of I/Q imbalance of the received signal in Figures 4-6 [2]. The transmitted baseband signal is shown in Figure 4. The signal is up-converted to RF and transmitted through the frequency-selective channel, resulting in the received RF signal depicted in Figure 5. Subsequently, the received signal is down-converted, because there exhibits I/Q imbalance, the mirror signal is not fully rejected, and mixes down into the regarded baseband channel in Figure 6. In the presence of I/Q imbalance, the carrier of mirror signal will affect the carrier of signal. The more mismatches of gain and phase, the more serious performance degradation I/Q imbalance brings, and bit error rate (BER) will also raise.
Preamble

We also know the effect of I/Q imbalance on the received signal by the constellation for SISO OFDM systems. Using 16-quadrature amplitude modulation (16-QAM) with the mismatches parameters $g=10\%$ and $\psi = 5^\circ$. The constellation with I/Q imbalance for SISO OFDM systems is shown in Figure 7. From Figure 7, it is shown that the 16 ideal red points become 16 dispersed points respectively. Assume 16 closed points are a group, and regard every group as a point, the 16 points are the result of the effect of the parameter $k_1$ and channel. The mirror signal leads to the 16 closed points in a group, it means that the interference of I/Q imbalance may lead the point be removed from original decision region. Thus, the wrong decision causes the wrong demodulation which will raise the BER.

III. PREAMBLE DESIGN AND ESTIMATION

A. Preamble Design Structure for the Transmitter

Consider the problem of jointly estimating the channel effect and I/Q imbalance, this paper proposes a preamble-aided estimation method. Referring the standard of IEEE 802.11n, there are four part in our novel preamble design shown on Figure 8. The four parts includes preamble sequence 1, preamble sequence 2, preamble sequence 3, and preamble sequence 4.

$$s_1(k), s_2(k) \in e^{j\varphi (k)} q^N \quad (q, N \text{ coprime})$$ \hspace{1cm} (6)

In order to estimate channel effect and the parameters of I/Q imbalance, the preamble design needs to involve the orthogonal property between the real part preambles and image part preambles. Therefore, the novel preamble design can assist the receiver to combat the image interference due to I/Q imbalance effect. Based on the requirement, the proposed novel preamble design structure, called full-usage preamble sequence design, is shown in Figure 9, which involves the conjugate symmetric preamble sequences and the different sign operation property to achieve the orthogonal feature. Besides, the Chu sequence [9] is utilized in the preamble to assist the proposed estimator with more accuracy performance, i.e.,

**Figure 8.** The structure of preamble design referred to IEEE 802.11n

**Figure 9.** The structure of full-usage preamble sequence design

B. Joint Estimation of I/Q Imbalance and Channel Effect

The overall schematic diagram of the proposed joint channel and I/Q imbalance parameter estimator is depicted in Figure 10 for MIMO OFDM systems. In the algorithm, we will first design the parameter estimator of SISO systems. Then, it can be extended to MIMO systems with more accuracy estimated performance.

**Figure 10.** Black diagram of joint I/Q imbalance and channel effect estimation of full-usage preamble sequence design
B.1. Estimators for SISO OFDM Systems

Using (4), the received signal in frequency domain without noise is

\[ y_m(k) = k_1 H(k) s(k) + k_2 H^\ast(-k) s^\ast(-k) \]  \hspace{8cm} (7)

where \( H(k) \) is given by

\[ H(k) = \begin{bmatrix} H_1(k) & 0 \\ 0 & H_2(k) \end{bmatrix} \]  \hspace{8cm} (8)

Then, using preamble 1 shown in Figure 9, (7) can be rewritten as

\[ y_1(k) = \begin{bmatrix} (k_1 H_1(k) + k_2 H_2^\ast(-k)) s_1(k) \\ (k_1 H_2(k) + k_2 H_1^\ast(-k)) s_1^\ast(-k) \end{bmatrix} \]  \hspace{8cm} (9)

Next, the upper half symbol of (9) is expressed by

\[ y_{11}(k) = [k_1 H_1(k) + k_2 H_2^\ast(-k)] s_1(k) \]  \hspace{8cm} (10)

For the bottom half symbol of (9), taking the complex conjugate at \(-k\) frequency of it is given by

\[ y_{12}^\ast(-k) = [k_1 H_1^\ast(-k) + k_2 H_2^\ast(-k)] s_1(k) \]  \hspace{8cm} (11)

Similarly, by preamble 2, we have

\[ y_2(k) = \begin{bmatrix} (k_1 H_1(k) - k_2 H_2^\ast(-k)) s_2(k) \\ (k_1 H_2(k) - k_2 H_1^\ast(-k)) s_2^\ast(-k) \end{bmatrix} \]  \hspace{8cm} (12)

\[ y_{21}(k) = [k_1 H_1(k) - k_2 H_2^\ast(-k)] s_2(k) \]  \hspace{8cm} (13)

\[ y_{22}^\ast(-k) = [-k_1 H_1^\ast(-k) + k_2 H_1^\ast(-k)] s_2(k) \]  \hspace{8cm} (14)

Referring to the property of the imbalance parameters in (15)-(16),

\[ k_1 + k_2^* = 1 \]  \hspace{8cm} (15)

\[ k_1 - k_2^* = g e^{\jmath \phi} = \mu \]  \hspace{8cm} (16)

Next, as shown in Figure 10, the first time-domain equal weight combiner technique of the two contiguous preambles in (10)-(11) and (13)-(14) is utilized to cancel the image interference due to the orthogonal preamble structure design, i.e.,

\[ v_1(k) = \frac{1}{2} (y_{11}(k) + y_{21}(k)) = k_1 H_1(k) s_1(k) \]  \hspace{8cm} (17)

\[ v_2(k) = \frac{1}{2} (y_{12}^\ast(-k) + y_{22}^\ast(-k)) = k_2 H_1(k) s_1(k) \]  \hspace{8cm} (18)

Furthermore, in order to estimate the CIR and imbalance parameters, the two separation techniques is proposed by the second frequency-domain equal weight combination and subtraction techniques of the two subcarriers symmetric preambles in (17) and (18), which is shown in Figure 10, i.e.,

\[ z_1(k) = v_1(k) + v_2(k) = H_1(k) s_1(k) \]  \hspace{8cm} (19)

\[ z_2(k) = v_1(k) - v_2(k) = \mu z_1(k) \]  \hspace{8cm} (20)

Hence, \( \hat{\mu} \) and \( \hat{H}_1(k) \) can be acquired by the cross correlation and auto correlation schemes of (19) and (20), i.e.,

\[ \hat{\mu}_1 = \frac{z_1^H(k) \cdot z_1(k)}{\|z_1(k)\|^2} \]  \hspace{8cm} (21)

\[ \hat{H}_1(k) = \text{diag} \left\{ z_1(k) \cdot s_1^\ast(k) \right\} \]  \hspace{8cm} (22)

Next, in order to enhance the estimation performance, the time-domain and frequency-domain weight combiner can be processed again for the image preambles, which is designed to cancel the real part preambles. Note that it is contrary to (17) and (18) algorithms. That is, from (10)-(11) and (13)-(14), we can get the another parameter estimated result, i.e.,

\[ v_3(k) = \frac{1}{2} (y_{11}(k) - y_{21}(k)) = k_2 H_2^\ast(-k) s_1(k) \]  \hspace{8cm} (23)

\[ v_4(k) = \frac{1}{2} (y_{12}^\ast(-k) - y_{22}^\ast(-k)) = k_2 H_2^\ast(-k) s_1(k) \]  \hspace{8cm} (24)

\[ z_3(k) = v_3^\ast(-k) + v_4^\ast(-k) = H_2(k) s_1^\ast(-k) \]  \hspace{8cm} (25)

\[ z_4(k) = v_3^\ast(-k) - v_4^\ast(-k) = \mu H_2(k) s_1^\ast(-k) \]  \hspace{8cm} (26)

\[ \hat{\mu}_2 = \frac{z_3^H(k) \cdot z_4(k)}{\|z_3(k)\|^2} \]  \hspace{8cm} (27)

\[ \hat{H}_2(k) = \text{diag} \left\{ z_3(k) \cdot s_1^\ast(-k) \right\} \]  \hspace{8cm} (28)

In order to obtain more accurate estimations via the law of average, the imbalance and CIR parameters are estimated as

\[ \hat{\mu} = (\hat{\mu}_1 + \hat{\mu}_2) / 2 \]  \hspace{8cm} (29)

\[ \hat{H}(k) = \text{diag} \left\{ \hat{H}_1(k), \hat{H}_2(k) \right\} \]  \hspace{8cm} (30)

B.2. Estimators for MIMO OFDM Systems

Next, the above proposed schemes can be extended to 2x2 MIMO OFDM systems. Figure 11 is shown the structure of channel between transmitter and receiver.

\begin{center}
Figure 11. The structure of channel for 2x2 MIMO OFDM systems
\end{center}

The channel can be written by

\[ H_{(N_t,N_r)}(k) = \begin{bmatrix} H_{11}^{(N_t,N_r)}(k) & 0 \\ 0 & H_{22}^{(N_t,N_r)}(k) \end{bmatrix} \]  \hspace{8cm} (31)

where \( N_t \) and \( N_r \) are denoted by the \( N_t \) th transmitted and the \( N_r \) th received antennas, respectively. As shown in Figure 10, on the basis of the previous algorithms in (17)-(22) of SISO OFDM systems and preamble sequences 1-4 in Figure 9,
Moreover, as shown in Figure 10, the I/Q imbalance parameters can be estimated by the cross and auto correlation schemes, i.e.,

\[
\hat{\mu}_i^{(i)} = \frac{\mathbf{z}_i^{(i)\H}(k) \cdot \mathbf{z}_i^{(i)(k)}}{\|\mathbf{z}_i^{(i)(k)}\|^2}, \text{ for } i = 1 \text{ and } 2
\]

\[
\hat{\mu}_i^{(i)} = \frac{\mathbf{z}_i^{(i)\H}(k) \cdot \mathbf{z}_i^{(i)(k)}}{\|\mathbf{z}_i^{(i)(k)}\|^2}, \text{ for } i = 1 \text{ and } 2
\]

Next, in order to estimate CIR, the combined signal in (32)-(39) can be rewritten by matrix form, i.e.,

\[
\begin{bmatrix}
\mathbf{z}_1^{(1)(k)} \\
\mathbf{z}_2^{(1)(k)} \\
\mathbf{z}_1^{(2)(k)} \\
\mathbf{z}_2^{(2)(k)}
\end{bmatrix} =
\begin{bmatrix}
\mathbf{s}_1(k) & \mathbf{s}_2(k) \\
-\mathbf{s}_2^\H(k) & \mathbf{s}_1^\H(k)
\end{bmatrix}
\begin{bmatrix}
\mathbf{H}_1^{(1,1)}(k) \\
\mathbf{H}_1^{(1,2)}(k)
\end{bmatrix}
\]

Thus, the CIR channel parameters can be estimated by preamble sequences via simple least square algorithm shown in Figure 10, which is only the scalar multiplication and division with low complexity benefit, i.e.,

\[
\begin{bmatrix}
\hat{\mathbf{H}}_1^{(1,1)}(k) \\
\hat{\mathbf{H}}_1^{(1,2)}(k)
\end{bmatrix} =
\begin{bmatrix}
\mathbf{s}_1(k) & \mathbf{s}_2(k) \\
-\mathbf{s}_2^\H(k) & \mathbf{s}_1^\H(k)
\end{bmatrix}^H
\begin{bmatrix}
\mathbf{z}_1^{(1)(k)} \\
-\mathbf{s}_2^\H(k) & \mathbf{s}_1^\H(k)
\end{bmatrix}
\frac{1}{\|\mathbf{s}_1(k)\|^2 + \|\mathbf{s}_2(k)\|^2}
\begin{bmatrix}
\mathbf{z}_1^{(1)(k)} \\
\mathbf{z}_2^{(1)(k)}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\hat{\mathbf{H}}_1^{(2,1)}(k) \\
\hat{\mathbf{H}}_1^{(2,2)}(k)
\end{bmatrix} =
\begin{bmatrix}
\mathbf{s}_1(k) & \mathbf{s}_2(k) \\
-\mathbf{s}_2^\H(k) & \mathbf{s}_1^\H(k)
\end{bmatrix}^H
\begin{bmatrix}
\mathbf{z}_1^{(2)(k)} \\
-\mathbf{s}_2^\H(k) & \mathbf{s}_1^\H(k)
\end{bmatrix}
\frac{1}{\|\mathbf{s}_1(k)\|^2 + \|\mathbf{s}_2(k)\|^2}
\begin{bmatrix}
\mathbf{z}_1^{(2)(k)} \\
\mathbf{z}_2^{(2)(k)}
\end{bmatrix}
\]

Similarly, in order to enhance the estimation performance, referring the SISO algorithms in (23)-(28), the another imbalance and channel parameters, i.e., \(\hat{\mathbf{H}}_2^{(1,1)}(k), \hat{\mathbf{H}}_2^{(1,2)}(k), \hat{\mathbf{H}}_2^{(2,1)}(k), \hat{\mathbf{H}}_2^{(2,2)}(k), \hat{\mu}_3^{(1)}, \hat{\mu}_4^{(1)}, \hat{\mu}_3^{(2)} \) and \(\hat{\mu}_4^{(2)}\), can be estimated by the image preambles of the time-frequency combiner scheme, which the derivations are omitted due to the same previous procedures. Moreover, by the law of average, the MIMO imbalance parameters can be acquired with more accurate estimation result, i.e.,

\[
\hat{\mu}_1 = \frac{\hat{\mu}_1^{(1)} + \hat{\mu}_2^{(1)} + \hat{\mu}_3^{(1)} + \hat{\mu}_4^{(1)}}{4}
\]

\[
\hat{\mu}_2 = \frac{\hat{\mu}_1^{(2)} + \hat{\mu}_2^{(2)} + \hat{\mu}_3^{(2)} + \hat{\mu}_4^{(2)}}{4}
\]

To sum of, based on the novel preamble structure design, a complete CIR and I/Q imbalance parameters estimator is proposed for SISO and MIMO OFDM systems. In next section, its performance will be confirmed by computer simulations.

**IV. SIMULATION RESULTS**

In this section, simulation results are conducted to demonstrate the performance of the proposed estimation algorithms. For all simulations, there are 64 subcarriers for SISO OFDM and 2×2 MIMO OFDM systems. There are preamble sequences and 6 OFDM symbols in a packet. The channel effect is Rayleigh fading channel. The preamble sequences are the Chu sequence and the transmission data is 16-QAM symbols. Assume I/Q imbalance only happen at received side. We consider three different mismatch parameter pair of I/Q imbalance cases: (1) g=1 and \(\psi = 0^\circ\), (2) g=1.1 and \(\psi = 5^\circ\), (3) g=1.1 and \(\psi = 10^\circ\). When g=1 and \(\psi = 0^\circ\), there is no I/Q imbalance in the systems.

![Figure 12. MSE of I/Q imbalance parameters for SISO and MIMO OFDM systems](image-url)
channel parameters can be accurately estimated as well. As expected, the proposed MIMO technique can achieve better performance than SISO systems.

![Figure 13. MSE of estimated channel for SISO and MIMO OFDM systems](image)

V. CONCLUSIONS

In this paper, the estimation techniques of the I/Q imbalance and multipath channel effects of SISO and MIMO OFDM system are proposed over the wireless Rayleigh fading channels. Note that a novel preamble structure design can provide the advantage of the low computational complexity to cancel the image interference and estimate the gain and phase parameters of I/Q imbalance and the channel impulse response (CIR). Simulation results confirm the proposed MIMO method can accurately estimate the CIR and I/Q imbalance parameters, and provide lower MSE performance than SISO methods over RF impairment and channel effect environments.

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