Capacity Analysis for Macro/Clustered Femto Coexisting Networks

Uk Jang, Hyunjin Lee and Keeseong Cho
Broadcasting & Telecommunications Convergence Research Laboratory
Electronics and Telecommunications Research Institute (ETRI), Daejeon, Korea.
E-mail: {ukjang, petrus, chokis}@etri.re.kr

***Abstract*— The randomly deployment of a femtocell has a critical effect on the performance of a macro-cell network due to cochannel interference. In our previous work, we are able to verify that the block diaginalization (BD) with an antenna selection algorithm over macro/femto coexisting network shows a better performance from the respective of both macro mobile station (MMS) and femto mobile station (FMS) utilizing the advantage of a multiple input multiple output (MIMO) system. In this paper, we describe the closed-form expression of capacity for both MMS and FMS over coexisting network. Through simulation, both MMS and FMS have a higher capacity gain due to the successive interference mitigation as compared with the case of selfish beamforming at femtocells. Besides, the outage probability of MUEs can be effectively maintained in the presence of a severe interference from femtocells.**

***Index Terms*— Block Diagonalization, Interference Mitigation, Capacity, Coexisting Networks**

I. INTRODUCTION

Femto-cells are viewed as a promising technique for mobile operations to improve indoor coverage and provide high data rate services in a cost effective manner in the 4^{th} generation networks and beyond [1]-[4]. Therefore, recent interference issues may arise in the downlink of coexisting macro/femto networks. In a downlink, each macro mobile station (MMS) suffers from strong interference from nearby femto-cells, which is a critical performance factor for MMSs. However, priority should generally be given to macro-cells rather than femto-cells. Therefore, the important aspect in coexisting network is that the performance of MMSs has to be maintained even though a large number of femto-cells may be deployed on top of a macro-cell. In order to address interference problems, recent researches have considered power control methods, interference mitigation techniques, and resource partitioning [5]-[7]. Unfortunately, most previous works on interference mitigation in coexisting networks focus on the case of each femto base station (FBS) being equipped with a single transmit antenna.

In this paper, we analyze the capacity of block diaginalization (BD) with an antenna selection algorithm proposed in our previous work [8]. We describe the closed-form expression of capacity for both MMS and femto mobile station (FMS) over macro-femto coexisting network. Through simulation, we are able to verify that the proposed algorithm shows a better performance from the respective of both MMS and FMS in terms of capacity gain and outage probability.

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II. SYSTEM MODEL FOR MACRO-FEMTO COEXISTING NETWORKS

Consider a multiuser downlink channel with K_M MMS and a single macro base station (MBS). Each MBS and MMS has N_T^M and N_R antennas, respectively. Closed-access F femtocells are randomly distributed over the cell boundary of the macro-cell coverage area. There are F FBS, among which each FBS has a single FMS. Each FBS and FMS has N_T^F and N_R antennas, respectively. For simplicity, co-channel interference from neighboring macro-cell transmissions is ignored.

A. Signal Model for Macro Mobile Station

To exploit the signal model for MMSs with BD, let $\mathcal{U}_M = \{1, \ldots, K_M\}$ denote a set of MMSs in a macro-cell. The transmit vector symbol of the k^{th} MMS is denoted by a vector $\mathbf{x}_{M,k} \in \mathbb{C}^{L_{M,k}\times 1}$. The received signal at the k^{th} MMS ($k \in \mathcal{U}_M$) is given by

$$\mathbf{y}_{M,k} = \mathbf{H}_{M,k}^{M}\mathbf{M}_{M,k}\mathbf{x}_{M,k} + \underbrace{\mathbf{H}_{M,k}^{M}\sum_{l\in\mathcal{U}_M, l\neq k}\mathbf{M}_{M,l}\mathbf{x}_{M,l}}_{\text{Inter MMS Interference}}$$
$$+ \underbrace{\sum_{c=1}^{C}\mathbf{H}_{M,k}^{c}\sum_{n\in\mathcal{U}_c}\mathbf{M}_{c,n}\mathbf{x}_{c,n}}_{\text{Interference from Clustered Femto-cell}} + \mathbf{n}_{M,k}$$

where

- $\mathbf{H}_{M,k}^{M} \in \mathbb{C}^{N_R\times N_T^M}$ denotes the channel matrix from the MBS to the k^{th} MMS.
- $\mathbf{n}_{M,k} \in \mathbb{C}^{N_R\times 1}$ is the addictive white Gaussian noise (AWGN) vector with zero mean and covariance matrix $\mathbb{E}(\mathbf{n}_{M,k}\mathbf{n}_{M,k}^H) = \sigma_n^2\mathbf{I}$.
- $\mathbf{M}_{M,k} \in \mathbb{C}^{N_T^M\times L_{M,k}}$ is a pre-coding matrix for the k^{th} MMS, which is a cascade of two pre-coding matrices $\mathbf{B}_{M,k}$ and $\mathbf{D}_{M,k}$ for BD, i.e., $\mathbf{M}_{M,k}$=$\mathbf{B}_{M,k}\mathbf{D}_{M,k}$, where $\mathbf{B}_{M,k}$ removes the inter-MMS interference and $\mathbf{D}_{M,k}$ is used for parallelizing and power allocation, where $\mathbb{E}(\mathbf{D}_{M,k}\mathbf{x}_{M,k}\mathbf{x}_{M,k}^H\mathbf{D}_{M,k}^H)$=$\mathbf{Q}_{M,k}$ is the transmit covariance matrix.
- C is the total number of clusters in a macro-cell.

- F_c is the total number of FBSs in the c^{th} cluster.
- $\mathcal{U}_c$ is the set of FMSs for the c^{th} cluster.
- $\mathbf{H}^c_{M,k} \in \mathbb{C}^{N_R \times F_c \cdot N_T^F}$ is the aggregate channel matrix from the c^{th} cluster to the k^{th} MMS. $\mathbf{H}^c_{M,k}$ =[$\mathbf{H}^{(1)}_{M,k}$, $\cdots$, $\mathbf{H}^{(f)}_{M,k}$], where $\mathbf{H}^{(f)}_{M,k} \in \mathbb{C}^{N_R \times N_T^F}$ is a channel matrix from the f^{th} FBS to the k^{th} MMS.
- $\mathbf{M}_{c,n} \in \mathbb{C}^{F_c \cdot N_T^F \times \widetilde{L}_{c,n}}$ is a precoding matrix for the n^{th} FMS in the c^{th} cluster, which is a cascade of two pre-coding matrices $\mathbf{B}_{c,k}$ and $\mathbf{D}_{c,k}$ for BD, i.e., $\mathbf{M}_{c,k}$=$\mathbf{B}_{c,k}\mathbf{D}_{c,k}$, where $\mathbf{B}_{c,k}$ removes the inter-FMS interference, and $\mathbf{D}_{c,k}$ is used for parallelizing and power allocation, where $\mathbb{E}(\mathbf{D}_{c,k}\mathbf{x}_{c,k}\mathbf{x}^H_{c,k}\mathbf{D}^H_{c,k})$=$\mathbf{Q}_{c,k}$ is the transmit covariance matrix. $\mathbf{M}_{c,n}$ =[$(\mathbf{M}^{(1)}_n)^H$, $(\mathbf{M}^{(2)}_n)^H$, $\cdots$, $(\mathbf{M}^{(f)}_n)^H$]^H, where $\mathbf{M}^{(f)}_n \in \mathbb{C}^{N_T^F \times \widetilde{L}_{c,n}}$ is a precoding matrix for the n^{th} FMS at the f^{th} FBS.
- $\mathbf{x}_{c,n} \in \mathbb{C}^{\widetilde{L}_{c,n} \times 1}$ is a transmitted vector for the n^{th} FMS in the c^{th} cluster.

B. Signal Model for Femto Mobile Station

In despite of MMS case, there are three types of interferences, 1) intra-femto interference, 2) interference from other clusters, and 3) interference from MBS. For antenna selection, we consider antenna selection matrices $\mathbf{R}^H_{c,k} \in \mathbb{R}^{\widetilde{L}_{c,k} \times N_R}$ that are formed by taking $\widetilde{L}_{c,k}$ rows from $\mathbf{I}_{N_R}$ [11], which means the k^{th} FMS selects $\widetilde{L}_{c,k}(\leq N_R)$ antennas p(or streams) to use. After antenna selection matrix $\mathbf{R}^H_{c,k}$ is applied to the received signal, the post-precessed received signal at the k^{th} FMS in the c^{th} cluster ($k \in \mathcal{U}_c$) is given by

$$\begin{aligned}\mathbf{y}_{c,k} = &\underbrace{\mathbf{R}^H_{c,k}\mathbf{H}^c_{c,k}\mathbf{M}_{c,k}\mathbf{x}_{c,k}}_{\text{Desired Signal}} + \underbrace{\mathbf{R}^H_{c,k}\mathbf{H}^c_{c,k}\sum_{l\neq k}\mathbf{M}_{c,l}\mathbf{x}_{c,l}}_{\text{Intra-cluster Interference}} \\ &+ \underbrace{\mathbf{R}^H_{c,k}\sum_{\hat{c}=1,\hat{c}\neq c}^{C}\mathbf{H}^{\hat{c}}_{c,k}\sum_{n\in\mathcal{U}_{\hat{c}}}\mathbf{M}_{\hat{c},n}\mathbf{x}_{\hat{c},n}}_{\text{Interference from other Clusters}} \\ &+ \underbrace{\mathbf{R}^H_{c,k}\mathbf{H}^M_{c,k}\sum_{m\in\mathcal{U}_{\mathcal{M}}}\mathbf{M}_{M,m}\mathbf{x}_{M,m}}_{\text{Interference from MBS}} + \mathbf{R}^H_{c,k}\mathbf{n}_{c,k} \qquad (1)\end{aligned}$$

where$\mathbf{H}^M_{c,k} \in \mathbb{C}^{N_R \times N_T^M}$ denotes the channel matrix from the MBS to the k^{th} FMS in the c^{th} cluster.

C. Block Diagonalization at MBS

In a downlink MIMO broadcast channel, BD is one of the solutions for canceling inter user interference [12][13]. We can define the aggregate interference channel matrix for selected MMS $k \in \mathcal{S}_M$ as $\widehat{\mathbf{H}}_{M,k}$ = $[(\mathbf{H}^M_{M,1})^H \ \ldots \ (\mathbf{H}^M_{M,k-1})^H, (\mathbf{H}^M_{M,k+1})^H \ldots \ (\mathbf{H}^M_{M,\widehat{K}})^H]^H$. In this case, the zero-interference constraint forces to lie in the null space of $\widehat{\mathbf{H}}_{M,k}$. Let us define the singular value decomposition (SVD) of $\widehat{\mathbf{H}}_{M,k}$ as $\widehat{\mathbf{H}}_{M,k}$ = $\widehat{\mathbf{U}}_{M,k}[\widehat{\mathbf{\Lambda}}_{M,k}\ \mathbf{0}_{\widehat{L}_{M,k}\times(N_T^M-\widehat{L}_{M,k})}][\widehat{\mathbf{V}}^{(1)}_{M,k}\ \ \widehat{\mathbf{V}}^{(0)}_{M,k}]^H$, where $\widehat{L}_{M,k}$ is the rank of $\widehat{\mathbf{H}}_{M,k}$, $\widehat{\mathbf{U}}_k$ is the left singular vector matrix of $\widehat{\mathbf{H}}_{M,k}$, $\widehat{\mathbf{\Lambda}}_{M,k}$ =diag($\lambda_{1,k}$, . . . , $\lambda_{\widehat{L}_{M,k},k}$) is the $\widehat{L}_{M,k} \times \widehat{L}_{M,k}$ diagonal matrix containing singular values. Matrices $\widehat{\mathbf{V}}^{(1)}_{M,k}$ and $\widehat{\mathbf{V}}^{(0)}_{M,k}$ denote the right singular matrices, each consisting of the singular vectors corresponding to the first $\widehat{L}_{M,k}$ non-zero singular values and the last $N_T^M - \widehat{L}_{M,k}$ zero singular values, respectively. Since the key idea of BD is that the columns of $\widehat{\mathbf{V}}^{(0)}_{M,k}$ form a null space basis of $\widehat{\mathbf{H}}_{M,k}$, we can choose the pre-coding matrix as $\mathbf{B}_{M,k} = \left(\widehat{\mathbf{V}}^{(0)}_{M,k}\right)_{(1:N_T^M-\widehat{L}_{M,k})}$. After inter-user-interference is perfectly canceled at the MBS, the effective channel of the k^{th} MMS after the BD process is $\mathbf{H}^{eff}_{M,k} = \mathbf{H}^M_{M,k}\mathbf{B}_{M,k} \in \mathbb{C}^{L_{M,k}\times L_{M,k}}$. Since the k^{th} MMS receives its own data stream without inter MMS interference, the methodology for designing an appropriate decoder is exactly the same as single-user MIMO cases, which means the SVD of $\mathbf{H}^{eff}_{M,k}$ is $\mathbf{H}^{eff}_{M,k}$=$\mathbf{U}_{M,k}\mathbf{\Lambda}_{M,k}\mathbf{V}^H_{M,k}$. We can take $\mathbf{D}_{M,k}$=$\mathbf{V}_{M,k}\mathbf{Q}^{\frac{1}{2}}_{M,k}$ where the $\mathbf{V}_{M,k}$ is the right singular vectors corresponding to non-zero singular values and $\mathbf{Q}^{\frac{1}{2}}_{M,k}$ denotes a diagonal matrix whose elements scale the power transmitted into each of the columns of $\mathbf{V}_{M,k}$. Finally, the aggregate pre-coder of the k^{th} MMS is given by $\mathbf{M}_{M,k} = \left(\widehat{\mathbf{V}}^{(0)}_{M,k}\right)_{(1:N_T^M-\widehat{L}_{M,k})}\mathbf{V}_{M,k}\mathbf{Q}^{\frac{1}{2}}_{M,k}$. There exist $N^I_{M,k}$ effective cochannel interferers from the clusters and the post-precessed received signal at the k^{th} MMS can be rewritten as

$$\mathbf{y}_{M,k} = \mathbf{H}^{eff}_{M,k}\mathbf{D}_{M,k}\mathbf{x}_{M,k} + I_{M,k} + \mathbf{n}_{M,k}$$
$$\text{where}\quad I_{M,k} = \underbrace{\sum_{c=1}^{C}\mathbf{H}^c_{M,k}\sum_{n\in\mathcal{U}_c}\mathbf{M}_{c,n}\mathbf{x}_{c,n}}_{\text{Interference from Clusters}}$$

where $I_{M,k} \in \mathbb{C}^{L_{M,k}\times 1}$ is the cochannel interference from clustered-FBSs.

III. Precoding Matrix Design at Clustered FBS

To mitigate the interferences from clustered FBSs to MMSs perfectly, the pre-coding matrices for the FMSs have to lie in the null space of the interference channel to the MBS.

To obtain precoding matrices that satisfy the null space constraint, each clustered-FBS stacks channels for the FMSs located at the c^{th} cluster ($\mathbf{R}^H_{c,k}\mathbf{H}^c_{c,k}, k \in \mathcal{S}_c$), and interfering channels for the n^{th} MMS located near the c^{th} cluster ($\mathbf{H}^c_{M,n}, n \in \mathcal{S}_M$), as follows:

$$\mathbf{H}_c = [(\mathbf{R}^H_{c,1}\mathbf{H}^c_{c,1})^H,\ \cdots,\ (\mathbf{R}^H_{c,K_c}\mathbf{H}^c_{c,K_c})^H,\ (\mathbf{H}^c_{M,n})^H]^H \qquad (2)$$

If rank($\mathbf{H}^c_{M,n}$) $= N_R, n \in \mathcal{S}_M$ i.e., $\rho = N_R$, the clustered-FBS uses $F_c \cdot N_T^F - N_R$ streams to serve the FMSs, and the number of $\rho = N_R$ degrees of freedom is used to cancel the interference to the MMS. The aggregate interference channel for the k^{th} FMS in the c^{th} cluster is

$$\begin{aligned}\widehat{\mathbf{H}}_{c,k} =&[(\mathbf{R}^H_{c,1}\mathbf{H}^c_{c,1})^H,\ \cdots,\ (\mathbf{R}^H_{c,k-1}\mathbf{H}^c_{c,k-1})^H,\ \cdots,\\ &(\mathbf{R}^H_{c,k+1}\mathbf{H}^c_{c,k+1})^H,\ \cdots,\ (\mathbf{R}^H_{c,K_c}\mathbf{H}^c_{c,K_c})^H,\ (\mathbf{H}^c_{M,n})^H]^H \qquad (3)\end{aligned}$$

If we apply SVD for $\widehat{\mathbf{H}}_{c,k}$ in (3), the aggregate interference channel is decomposed as $\widehat{\mathbf{H}}_{c,k}$ =

$\widehat{\mathbf{U}}_{c,k}[\widehat{\mathbf{\Lambda}}_{c,k} \ \ \mathbf{0}_{\overline{L}_{c,k}\times(F_C N_T^F-\overline{L}_{c,k})}][\widehat{\mathbf{V}}_{c,k}^{(1)} \ \ \widehat{\mathbf{V}}_{c,k}^{(0)}]^H$, where $\overline{L}_{c,k}$ is the rank of $\widehat{\mathbf{H}}_{c,k}$, $\widehat{\mathbf{U}}_{c,k}$ is the left singular vector matrix of $\widehat{\mathbf{H}}_{c,k}$ and $\widehat{\mathbf{\Lambda}}_{c,k}$ =diag($\lambda_{1,k}$, ... , $\lambda_{\overline{L}_{c,k},k}$) is the $\overline{L}_{c,k} \times \overline{L}_{c,k}$ diagonal matrix containing singular values. Matrices $\widehat{\mathbf{V}}_{c,k}^{(1)}$ and $\widehat{\mathbf{V}}_{c,k}^{(0)}$ denote right singular matrices consisting of singular vectors corresponding to the first $\overline{L}_{c,k}$ non-zero singular values and last $F_C N_T^F - \overline{L}_{c,k}$ zero singular values, respectively. Since the key idea of BD is that the columns of $\widehat{\mathbf{V}}_{c,k}^{(0)}$ form a null space basis of $\widehat{\mathbf{H}}_{c,k}$, we can choose the pre-coding matrix $\mathbf{B}_{c,k}$ as

$$\mathbf{B}_{c,k} = \left(\widehat{\mathbf{V}}_{c,k}^{(0)}\right)_{(1:F_c N_T^F - \overline{L}_{c,k})}.$$

After inter-FMS interference is perfectly canceled, the effective channel of the k^{th} FMS after BD process is $\mathbf{H}_{c,k}^{eff} = \mathbf{R}_{c,k}^H \mathbf{H}_{c,k}^c \mathbf{B}_{c,k} \in \mathbb{C}^{\widetilde{L}_{c,k}\times\widetilde{L}_{c,k}} = \mathbf{U}_{c,k}\mathbf{\Lambda}_{c,k}\mathbf{V}_{c,k}^H$. We can take $\mathbf{D}_{c,k} = \mathbf{V}_{c,k}\mathbf{Q}_{c,k}^{\frac{1}{2}}$ where $\mathbf{V}_{c,k}$ is the right singular vectors corresponding to non-zero singular values, and $\mathbf{Q}_{c,k}^{\frac{1}{2}}$ denotes a diagonal matrix whose elements scale the power transmitted into each column of $\mathbf{V}_{c,k}$. Finally, the aggregate pre-coder of the k^{th} FMS $\mathbf{M}_{c,k}$ is given by

$$\mathbf{M}_{c,k} = \left(\widehat{\mathbf{V}}_{c,k}^{(0)}\right)_{(1:F_c N_T^F - \overline{L}_{c,k})} \mathbf{V}_{c,k}\mathbf{Q}_{c,k}^{\frac{1}{2}}.$$

The received signal of the k^{th} FMS $\mathbf{y}_{c,k}$ in (1) is rewritten as,

$$\mathbf{y}_{c,k} = \mathbf{H}_{c,k}^{eff}\mathbf{D}_{c,k}\mathbf{x}_{c,k} + \mathbf{R}_{c,k}^H\left(I_{c,k} + \mathbf{n}_{c,k}\right),$$

$$\text{where } \ I_{c,k} = \underbrace{\sum_{\hat{c}=1,\hat{c}\neq c}^{C} \mathbf{H}_{c,k}^{\hat{c}} \sum_{n\in\mathcal{U}_{\hat{c}}} \mathbf{M}_{\hat{c},n}\mathbf{x}_{\hat{c},n}}_{\text{Interference from other Clusters}} + \underbrace{\mathbf{H}_{c,k}^{M} \sum_{m\in\mathcal{U}_{\mathcal{M}}} \mathbf{M}_{M,m}\mathbf{x}_{M,m}}_{\text{Interference from MBS}}$$

where $I_{c,k} \in \mathbb{C}^{\widetilde{L}_{c,k}\times 1}$ is the cochannel interferences from other clusters and the MBS.

IV. Capacity Analysis for Coexisting Network

A. Capacity of MMS

1) Capacity 1 (without Femto-cells): There are no interferences from femto-cells since there are no femto-cells. The capacity for the k^{th} MMS is given as,

$$\mathcal{C}_{M,k}^1 = \log_2 \det\left|\mathbf{I}_{N_R} + \mathbf{H}_{M,k}^{eff}\mathbf{Q}_{M,k}(\mathbf{H}_{M,k}^{eff})^H\right| \tag{4}$$

2) Capacity 2 (Proposed Scheme with Clustered Femto): In proposed scheme, when each clustered-FBS generates a precoding matrix, the c^{th} clustered-FBS considers the selected interfering channel to the $(k^*)^{th}$ MMS, $\mathbf{H}_{c,k^*}$. Let Ω_k be the set of clusters that select $\mathbf{H}_{c,k}$, i.e.,

$$\Omega_k = \{c \mid k = \arg_{k^*} \mathbf{H}_c(k^*)\} \ \text{ and } \ |\Omega_k| = \widehat{C}, \tag{5}$$

where $\mathbf{H}_c(k^*)$ is defined in [8].

At the k^{th} MMS, the interferences from Ω_k are mitigated effectively due to the proposed precoding matrix design with the antenna selection at Ω_k. However, since there are $N_{M,k}^I$ effective cochannel interferers from the $\Omega_j, j \neq k$ clusters, the effective cochannel interference signal at the k^{th} MMS is

$$I_{M,k} = \sum_{c\in\Omega_j, j\neq k} \mathbf{H}_{c,k} \sum_{n\in\mathcal{U}_c} \mathbf{M}_{c,n}\mathbf{x}_{c,n} \tag{6}$$

The covariance matrix of (6) is given by

$$\begin{aligned}\mathbf{\Gamma}_{M,k}^2 &= \sum_{c\in\Omega_j, j\neq k}\sum_{n\in\mathcal{U}_c} \mathbf{H}_{c,k}\mathbf{B}_{c,n}\mathbb{E}(\mathbf{D}_{c,n}\mathbf{x}_{c,n}\mathbf{x}_{c,n}^H\mathbf{D}_{c,n}^H)\mathbf{B}_{c,n}^H\mathbf{H}_{c,k}^H \\ &= \sum_{c\in\Omega_j, j\neq k}\sum_{n\in\mathcal{U}_c} \mathbf{H}_{c,k}\mathbf{B}_{c,n}\mathbf{Q}_{c,n}\mathbf{B}_{c,n}^H\mathbf{H}_{c,k}^H \end{aligned} \tag{7}$$

Denote $\mathbf{\Phi}_{M,k}^2 = \mathbf{\Gamma}_{M,k}^2 + \sigma_n^2\mathbf{I}_{N_R}$ as the covariance matrix of the sum of interferences from clusters plus noise, and the capacity of the k^{th} MMS is then given as,

$$\mathcal{C}_{M,k}^2 = \log_2 \det\left|\mathbf{I}_{N_R} + \mathbf{H}_{M,k}^{eff}\mathbf{Q}_{M,k}(\mathbf{H}_{M,k}^{eff})^H(\mathbf{\Phi}_{M,k}^2)^{-1}\right| \tag{8}$$

3) Capacity 3 (Selfish BD at Femto-cells): In the presence of femto-cells in a macro-cell, the effective cochannel interference signal at the k^{th} MMS is

$$I_{M,k} = \sum_{c=1}^{C} \mathbf{H}_{c,k} \sum_{n\in\mathcal{U}_c} \mathbf{M}_{c,n}\mathbf{x}_{c,n} \tag{9}$$

The covariance matrix of (9) is given by

$$\begin{aligned}\mathbf{\Gamma}_{M,k}^3 &= \sum_{c=1}^{C}\sum_{n\in\mathcal{U}_c} \mathbf{H}_{c,k}\mathbf{B}_{c,n}\mathbb{E}(\mathbf{D}_{c,n}\mathbf{x}_{c,n}\mathbf{x}_{c,n}^H\mathbf{D}_{c,n}^H)\mathbf{B}_{c,n}^H\mathbf{H}_{c,k}^H \\ &= \sum_{c=1}^{C}\sum_{n\in\mathcal{U}_c} \mathbf{H}_{c,k}\mathbf{B}_{c,n}\mathbf{Q}_{c,n}\mathbf{B}_{c,n}^H\mathbf{H}_{c,k}^H \end{aligned} \tag{10}$$

Denote $\mathbf{\Phi}_{M,k}^3 = \mathbf{\Gamma}_{M,k}^3 + \sigma_n^2\mathbf{I}_{N_R}$ as the covariance matrix of the sum of interferences from clusters plus noise, and the capacity of the k^{th} MMS is then given as

$$\mathcal{C}_{M,k}^3 = \log_2 \det\left|\mathbf{I}_{N_R} + \mathbf{H}_{M,k}^{eff}\mathbf{Q}_{M,k}(\mathbf{H}_{M,k}^{eff})^H(\mathbf{\Phi}_{M,k}^3)^{-1}\right| \tag{11}$$

B. Capacity of FMS

1) Capacity 4 (Selfish BD at Femto-cells): If each femto-cell operates in a selfish manner, each FMS suffers strong interferences: 1) MMS interference and 2) inter-femto interference. The received signal at the k^{th} FMS in the f^{th} femto-cell ($k \in \mathcal{U}_f$) is

$$\mathbf{y}_k^f = \mathbf{H}_k^{(f)}\mathbf{M}_k^{(f)}\mathbf{x}_k^{(f)} + I_k^{(f)} + \mathbf{n}_k \tag{12}$$

where

$$I_k^{(f)} = \underbrace{\sum_{l\neq f}\mathbf{H}_k^{(l)}\sum_{n\in\mathcal{U}_l}\mathbf{M}_n^{(l)}\mathbf{x}_n^{(l)}}_{\text{Inter-femto Interference}} + \underbrace{\mathbf{H}_{M,k}\sum_{m\in\mathcal{U}_{\mathcal{M}}}\mathbf{M}_{M,m}\mathbf{x}_{M,m}}_{\text{Interference from MBS}} \tag{13}$$

The covariance matrix of (13) is given by

$$\begin{aligned}\mathbf{\Gamma}_k^{(f)} &= \sum_{l\neq f}\sum_{n\in\mathcal{U}_l} \mathbf{H}_k^{(l)}\mathbf{M}_n^{(l)}\mathbb{E}(\mathbf{x}_n^{(l)}(\mathbf{x}_n^{(l)})^H)(\mathbf{M}_n^{(l)})^H(\mathbf{H}_k^{(l)})^H \\ &+ \sum_{m\in\mathcal{U}_\mathcal{M}} \mathbf{H}_{M,k}\mathbf{B}_{M,m}\mathbb{E}(\mathbf{D}_{M,m}\mathbf{x}_{M,m}\mathbf{x}_{M,m}^H\mathbf{D}_{M,m}^H)\mathbf{B}_{M,m}^H\mathbf{H}_{M,k}^H \\ &= \sum_{l\neq f}\sum_{n\in\mathcal{U}_l} \mathbf{H}_k^{(l)}\mathbf{M}_n^{(l)}\mathbf{Q}_n^{(l)}(\mathbf{M}_n^{(l)})^H(\mathbf{H}_k^{(l)})^H \\ &+ \sum_{m\in\mathcal{U}_\mathcal{M}} \mathbf{H}_{M,k}\mathbf{B}_{M,m}\mathbf{Q}_{M,m}\mathbf{B}_{M,m}^H\mathbf{H}_{M,k}^H \end{aligned} \tag{14}$$

Denote $\mathbf{\Phi}_k^{(f)} = \mathbf{\Gamma}_k^{(f)} + \sigma_n^2\mathbf{I}_{N_R}$ as the covariance matrix of the sum of interferences from femto-cells plus noise, and the capacity of the k^{th} FMS in the f^{th} femto-cell is then given as

$$\mathcal{C}_k^{(f)} = \log_2\det\left|\mathbf{I}_{N_R} + \mathbf{H}_k^{(eff,f)}\mathbf{Q}_k^{(f)}\left(\mathbf{H}_k^{(eff,f)}\right)^H(\mathbf{\Phi}_k^{(f)})^{-1}\right| \tag{15}$$

2) *Capacity 5 (Proposed Clustered Femto):* If femto-cells form clusters using the proposed algorithm, the inter-femto interference will be perfectly canceled, which means all propagation links (including interfering channels) are exploiting useful data. Unfortunately, the inter-cluster interference will remain if all clusters operates at the same frequency. To reduce the inter-cluster interference, we are able to consider the fractional frequency reuse (FFR), which is when each cluster uses a different frequency band.

Case 1 : No FFR

If each cluster operates at the same frequency, inter-cluster interference exists. Therefore, the effective cochannel interference signal at the k^{th} FMS is

$$I_{c,k} = \underbrace{\sum_{\hat{c}=1,\hat{c}\neq c}^{C}\mathbf{H}_{\hat{c},k}\sum_{n\in\mathcal{U}_{\hat{c}}}\mathbf{M}_{\hat{c},n}\mathbf{x}_{\hat{c},n}}_{\text{Interference from other Clusters}} + \underbrace{\mathbf{H}_{M,k}\sum_{m\in\mathcal{U}_\mathcal{M}}\mathbf{M}_{M,m}\mathbf{x}_{M,m}}_{\text{Interference from MBS}} \tag{16}$$

The covariance matrix of (16) is given by

$$\begin{aligned}\mathbf{\Gamma}_{c,k}^2 &= \sum_{\hat{c}=1,\hat{c}\neq c}^{C}\sum_{n\in\mathcal{U}_{\hat{c}}}\mathbf{H}_{\hat{c},k}\mathbf{B}_{\hat{c},n}\mathbb{E}(\mathbf{D}_{\hat{c},n}\mathbf{x}_{\hat{c},n}\mathbf{x}_{\hat{c},n}^H\mathbf{D}_{\hat{c},n}^H)\mathbf{B}_{\hat{c},n}^H\mathbf{H}_{\hat{c},k}^H \\ &+ \sum_{m\in\mathcal{U}_\mathcal{M}}\mathbf{H}_{M,k}\mathbf{B}_{M,m}\mathbb{E}(\mathbf{D}_{M,m}\mathbf{x}_{M,m}\mathbf{x}_{M,m}^H\mathbf{D}_{M,m}^H)\mathbf{B}_{M,m}^H\mathbf{H}_{M,k}^H \\ &= \sum_{\hat{c}=1,\hat{c}\neq c}^{C}\sum_{n\in\mathcal{U}_{\hat{c}}}\mathbf{H}_{\hat{c},k}\mathbf{B}_{\hat{c},n}\mathbf{Q}_{\hat{c},n}\mathbf{B}_{\hat{c},n}^H\mathbf{H}_{\hat{c},k}^H \\ &+ \sum_{m\in\mathcal{U}_\mathcal{M}}\mathbf{H}_{M,k}\mathbf{B}_{M,m}\mathbf{Q}_{M,m}\mathbf{B}_{M,m}^H\mathbf{H}_{M,k}^H \end{aligned} \tag{17}$$

Denote $\mathbf{\Phi}_{c,k}^2 = \mathbf{\Gamma}_{c,k}^2 + \sigma_n^2\mathbf{I}_{\widetilde{L}_{c,k}}$ as the covariance matrix of the sum of interferences from femto-cells plus noise, and the capacity of the k^{th} FMS in the c^{th} cluster is then given as

$$\mathcal{C}_{c,k}^2 = \log_2\det\left|\mathbf{I}_{\widetilde{L}_{c,k}} + \mathbf{H}_{c,k}^{eff}\mathbf{Q}_{c,k}\left(\mathbf{H}_{c,k}^{eff}\right)^H(\mathbf{\Phi}_{c,k}^2)^{-1}\right| \tag{18}$$

Case 2 : FFR

If each cluster operates at a different frequency, the inter-cluster interference will be perfectly canceled. Therefore, the effective cochannel interference signal at the k^{th} FMS is

$$I_{c,k} = \underbrace{\mathbf{H}_{M,k}\sum_{m\in\mathcal{U}_\mathcal{M}}\mathbf{M}_{M,m}\mathbf{x}_{M,m}}_{\text{Interference from MBS}} \tag{19}$$

Since each cluster uses a part of the frequency, whole frequency channels are divided into C subchannels. The covariance matrix of (19) is given by

$$\begin{aligned}&\mathbf{\Gamma}_{c,k}^3 \\ &= \sum_{m\in\mathcal{U}_\mathcal{M}}\mathbf{H}_{M,k}\mathbf{B}_{M,m}\mathbb{E}(\mathbf{D}_{M,m}\mathbf{x}_{M,m}\mathbf{x}_{M,m}^H\mathbf{B}_{M,m}^H)\mathbf{D}_{M,m}^H\mathbf{H}_{M,k}^H \\ &= \sum_{m\in\mathcal{U}_\mathcal{M}}\mathbf{H}_{M,k}\mathbf{B}_{M,m}\mathbf{Q}_{M,m}\mathbf{B}_{M,m}^H\mathbf{H}_{M,k}^H \end{aligned} \tag{20}$$

Denote $\mathbf{\Phi}_{c,k}^3 = \mathbf{\Gamma}_{c,k}^3 + \sigma_n^2\mathbf{I}_{\widetilde{L}_{c,k}}$ as the covariance matrix of the sum of interferences from femto-cells plus noise, and the capacity of the k^{th} FMS in the c^{th} cluster is then given as

$$\mathcal{C}_{c,k}^3 = \frac{1}{C}\log_2\det\left|\mathbf{I}_{\widetilde{L}_{c,k}} + \mathbf{H}_{c,k}^{eff}\mathbf{Q}_{c,k}\left(\mathbf{H}_{c,k}^{eff}\right)^H(\mathbf{\Phi}_{c,k}^3)^{-1}\right| \tag{21}$$

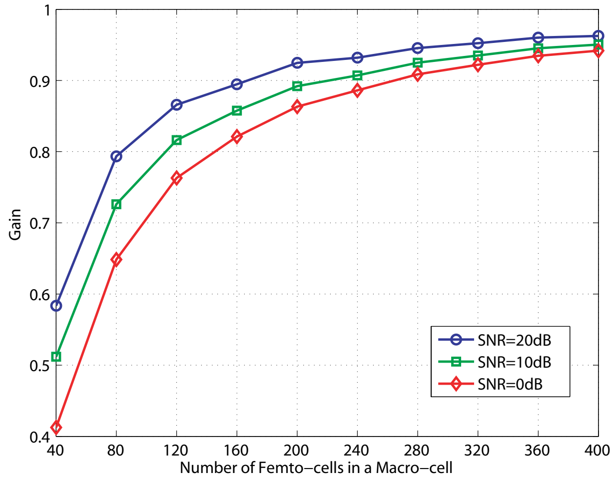

Fig. 1. Capacity Gain.

C. Capacity Gain

From the simulation results in [8], we can conclude that clustering with FFR is the best solution from the perspective for capacity of both MMSs and FMSs. Now, we define the capacity gain of the proposed algorithm as

$$G = \frac{\mathcal{C}_{pro} - \mathcal{C}_{ord}}{\mathcal{C}_{pro}}. \tag{22}$$

where

$$\mathcal{C}_{pro} = \sum_{k\in\mathcal{U}_M}\mathcal{C}_{M,k}^2 + \sum_{c=1}^{C}\sum_{k\in\mathcal{U}_c}\mathcal{C}_{c,k}^3. \tag{23}$$

$$\mathcal{C}_{ord} = \sum_{k\in\mathcal{U}_M}\mathcal{C}_{M,k}^3 + \sum_{f=1}^{F}\sum_{k\in\mathcal{U}_f}\mathcal{C}_k^{(f)} \tag{24}$$

Figure 1 plots the capacity gain of the proposed scheme versus the total number of femto-cells in a macro-cell according to

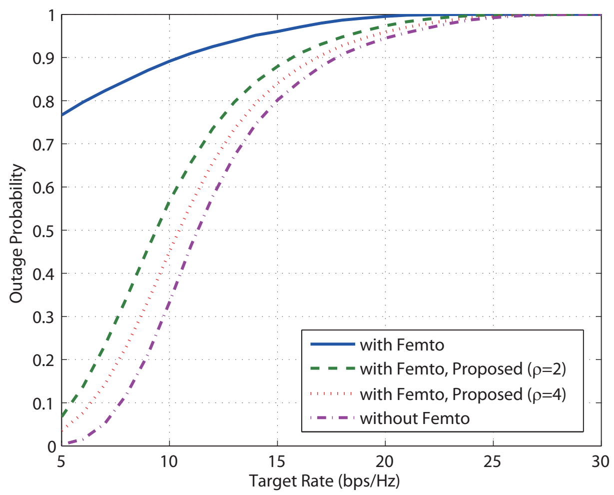

Fig. 2. Outage Probability of MUE with SNR=20dB, $N_R = 2$ and $F = 40$. As ρ increases, the performance of coexisting networks approaches to that of non-coexisting networks.

the different SNR. As the number of femto-cells increases, the performance gain also increases regardless of SNR levels. This is not surprising since all of the users (both MMSs and FMSs) have a higher capacity due to the successive interference mitigation as compared with the case of selfish beamforming.

D. Outage Probability

The outage probabilities of MUEs are defined as the probability that the channel cannot support a given target rate R_{Tar},

$$P_{out}^1 = P(\mathcal{C}_{M,k}^1 < R_{Tar})$$
$$P_{out}^2 = P(\mathcal{C}_{M,k}^2 < R_{Tar})$$
$$P_{out}^3 = P(\mathcal{C}_{M,k}^3 < R_{Tar})$$

where P_{out}^1, P_{out}^2 and P_{out}^3 represent the outage probability of FMUs for different system environments related to (4), (8) and (11), respectively. Fig. 2 shows that the outage probabilities of MUEs for the conventional without femto-cells, conventional with femto-cells and proposed with femto-cells. For a given target rate, it is seen that the proposed algorithm is very effective to achieve the reliable transmission against interference-heavy macro-femto coexisting environment. If each cluster uses the large number of degrees of freedom ($\rho = 4$) to suppress the interference to MUEs, it can be effectively suppressed, i.e., the performance of MUE increases as ρ increases. As a result, we can expect that the performance of the proposed algorithm approaches to the case of non-coexisting networks as ρ increases. This result has a great significance since the performance of MUEs can be effectively maintained using our proposed algorithm in the presence of a severe interference from femto-cells.

V. Conclusion

In this paper, we analyze the capacity of BD with an antenna selection algorithm. Based on our previous work, we describe the closed-form expression of capacity for both MMS and FMS over macro-femto coexisting network. Through numerical analysis and simulation, we can show that the capacity gain and outage probability of both MMS and FMS can be guaranteed effectively by performing clustering-based antenna selection and beamformer design at clustered-FBSs in a cooperative fashion.

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