The Optimum Ring Ratio of 16-APSK in LTE Uplink over Nonlinear System

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Abstract—Single Carrier Frequency Division Multiple Access (SC-FDMA) has been selected for use in the Long Term Evolution (LTE) uplink due to its lower Peak-to-Average Power Ratio (PAPR) relative to OFDMA. The resultant lower PAPR results in fewer excursions into the amplifier’s nonlinear region, where signal distortion can occur and results in degraded bit error rate (BER). The SC-FDMA scheme normally applies 16-ary Quadrature Amplitude Modulation (16-QAM), but amplitude phase shift keying (APSK) modulation has a lower PAPR than does 16-QAM, resulting in improved BER. This paper investigates the constellation ring ratio of the 16-APSK modulation scheme and its effects on BER through its effects on the PAPR. Simulation results are used to conclude that a ring ratio that ranges from 2.5 to 3.5 delivers the best results and provides BER and PAPR improvement.

Keyword—16-APSK, SC-FDMA, LTE, Nonlinear system, PAPR

I. INTRODUCTION

The 3GPP Long Term Evolution or LTE is a standard that has been developed by the 3rd Generation Partnership Project (3GPP), to become a truly global mobile standard and to apply to high-speed wireless data transmission for mobile and data terminal [1], [2]. It is the next step in progression from GSM / EDGE and UMTS / HSPA technologies with higher capacity and faster speed.

In order to reduce the interference signals and to improve the channel capacity of the LTE system, orthogonal frequency division multiple access (OFDMA) and single carrier frequency multiple access (SC-FDMA) are employed for the downlink and uplink, respectively. The SC-FDMA is the multiplexing technique for the LTE uplink because its PAPR is lowers than that of other transmission techniques, especially, OFDMA [1], [3], [4].

The high PAPR is a principal weakness of OFDM. It brings on the signal distortion in the nonlinear region of high power amplifier (HPA) [3]. The closer the HPA is operated to the saturation region, the more frequent are excursions into the nonlinear region and the more pronounced is the signal distortion. The signal distortion induces the degradation of bit error rate (BER). Therefore, it is desirable to have low PAPR to improve the BER performance.

To reduce the PAPR of OFDM, there are mainly two categories, which are signal scrambling techniques and signal distortion techniques [5]. The practical solutions of the signal scrambling techniques include block coding, Selective Level Mapping (SLM), and Partial Transmit Sequences (PTS). The signal distortion techniques include clipping, peak windowing, peak cancellation, peak power suppression, weighted multicarrier transmission, companding, etc.

Previously, there have been many researches on the PAPR reduction techniques. For examples, the signal scrambling techniques of the Partial Transmit Sequences and the block coding are proposed in [6] and [11]. The signal distortion techniques of clipping, suppression, peak windowing, peak cancellation, and companding are proposed in [7], [8], [9], [10], and [12], respectively. However, the disadvantage of all these techniques is the complexity increase of the system.

Other than the PAPR reduction techniques stated above, J. Gazda, D. Dupak and D. Kocur in [4] proposed to apply the M-ary APSK modulation to reduce the PAPR in the LTE uplink. They show that 16-APSK outperforms the standardized 16-QAM, which is generally used in the LTE uplink. The BER performance of the 16-APSK system improves for two reasons, the low PAPR of the APSK modulation causes fewer excursions into the nonlinear region and the 16-APSK signal is more robust to the distortions caused by the amplifier nonlinearities than is 16-QAM. Also, W. Sung, S. Kang, P. Kim and D. Chang in [13] give the preliminary analysis on the PAPR of the 16-APSK modulation in the digital video broadcasting for satellites (DVB-S2). They show that the PAPR of the 16-APSK depends on a ratio of the radius of each ring.

However, according to the authors’ knowledge, the effects of the constellation ring ratio of the 16-APSK (4+12 APSK) modulation scheme are not yet evaluated in the LTE uplink system. Therefore, this paper will present the investigation on the constellation ring ratio of the 16-APSK modulation used in the LTE uplink system with the nonlinear distortion due to a HPA. The optimal value of the ring ratio is also proposed.
The paper is organized as follows. First, the effects of the constellation ring ratio of the 16-APSK modulation, on both the PAPR and the BER, are presented in Section II. Section III shows the description of the system model adopted for the analysis and the simulation. Then, the simulation results are shown in section IV. Finally, section V provides conclusions.

II. EFFECTS OF THE APSK CONSTELLATION RING RATIO

Amplitude phase shift keying (APSK) is a digital modulation scheme that conveys data by changing or modulating both the amplitude and the phase. It is a combination of Amplitude shift keying (ASK) and Phase shift keying (PSK) applied to more effectively populate the signal space. Its constellation is composed of concentric rings, each with uniformly spaced PSK points [14]. Fig. 1 shows an example of 16-APSK constellation which has 2 concentric rings, with 4 PSK points in the inner and 12 PSK points in the outer. The effects of its constellation ring ratio on PAPR and BER could be analyzed as following.

A. The Effects on PAPR

Peak to Average Power Ratio (PAPR) is the ratio between the maximum power and the average power of the transmitted signal. The PAPR is defined as [3]

\[
PAPR = \max_{n=0,1,...,N-1} \left( \frac{1}{N} \sum_{n=0}^{N-1} |\hat{x}_n|^2 \right)
\]

where \( \hat{x}_n \) is the transmitted signal.

For 16-APSK modulation, there are many types of constellation patterns, such as 4+12 ASPK, 5+11 ASPK, and 6+10 APSK. Each type assigns a different number of points to each ring, with the total number of points summing to 16. Here, the 4+12 APSK is used in the investigation, since it delivers the best performance under the nonlinear conditions imposed by the saturation region of a power amplifier and is adopted in DVB-S2 [15].

The average power level of the 4+12 APSK, which has the constellation mapping shown in Fig. 1, can be defined as [13]

\[
E_s = \frac{R_i^2 + 3R_o^2}{4} = \frac{(1+3\beta_o)R_i^2}{4}
\]

where \( R_i \) is the radius of the inner ring,

\( R_o \) is the radius of the outer ring,

and \( \beta_o = R_o / R_i \) is the ratio between the outer and the inner ring radius.

From eq. (1) and (2), we can plot the relationship between the ratio (R2/R1) of the 4+12 APSK and the PAPR as shown in Fig. 2. This shows that the PAPR increases when the ratio (R2/R1) increases.

B. The Effects on BER

Considering the 16-APSK modulation for linear transmitter over an additive white Gaussian noise (AWGN) channel operating at energy per bit to noise power spectral density ratio (Eb/No) of 5 dB and 6 dB, the simulation results in Fig. 3 show the effects of the ratio (R2/R1) on the performance of the system. From Fig. 3, it is seen that the ratio in the range from 2.5 to 5 gives the lowest BER for both Eb/No values.
From the effects of the constellation ring ratio of 16-APSK modulation on the PAPR and BER, the appropriate value of the ratio can be determined by selecting the ratio that minimizes both the PAPR and the BER.

The previous analysis shows the effects of the constellation ring ratio of 16-APSK modulation on the PAPR in general and the BER for the linear channel only. This clearly shows that the constellation ring ratio can play a significant role in the nonlinear channel as well.

Next, the optimal ratio values providing a low BER and PAPR in the presence of nonlinearities will be addressed.

III. SYSTEM MODEL

Fig. 4 shows the block diagram of the general SC-FDMA system [3] with the APSK modulation. The binary sequence is modulated into data symbols, C(n), by using the 4+12 APSK modulation scheme. Then, a block of N modulated symbols is applied to the N-point Discrete Fourier Transform (DFT) to produce a frequency domain representation, D(k).

\[
D(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} C(n) e^{-j2\pi nk/N} \quad (3)
\]

This output is fed to the Localized FDMA (LFDMA) subcarrier mapping block, which is the subcarrier mapping used in the LTE standard. At this point, each of the N-DFT outputs is mapped to one of M orthogonal subcarriers (M>N) as

\[
S(k) = \begin{cases} 
D(k) & 0 \leq k \leq M-1 \\
0 & \text{Otherwise} 
\end{cases} \quad (4)
\]

Then, the M-point inverse DFT (IDFT) transforms these data symbols into the time-domain samples, T(t), of these subcarriers.

\[
T(t) = \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} S(k) e^{j2\pi mk/M} 
\]

Similar to OFDM, the cyclic prefix is added to the symbols. Before entering the AWGN channel, the transmitted signal is amplified by the high power amplifier (HPA), which makes the overall channel nonlinear. The signal at this point can be expressed as

\[
V(t) = A[r(t)]e^{j[\Phi(r(t)) + \theta(t)]} \quad (5)
\]

where r(t) and \( \theta(t) \) are the amplitude and phase of the input signal to the HPA.

At the receiver, the received signal is processed by removing the cyclic prefix, first. Then, the signals are demodulated by using M-point DFT, subcarrier De-mapping, N-point IDFT and the APSK demodulator.

For this investigation, the nonlinear HPA uses the Soft Limiter (SL) model, which defines the Amplitude Modulation to Amplitude Modulation (AM/AM) and the Amplitude Modulation to Phase Modulation (AM/PM) characteristics as following [8]

\[
F[\rho] = \begin{cases} 
-A, & \text{if } \rho < -A \\
\rho, & \text{if } -A \leq \rho \leq A \\
A, & \text{if } \rho > A 
\end{cases} \quad (6)
\]

and

\[
\Phi[\rho] = 0 \quad (7)
\]

where \( A \) is the saturating level of the amplifier, \( \rho \) is the amplitude of the input signals, \( F[\rho] \) is the amplitude of the output signals of the Soft Limiter, and \( \Phi[\rho] \) is the phase of the output signals of the Soft Limiter.

There is only amplitude distortion and no phase distortion in the SL model. The severity of the distortion can be measured by the Input-Back-Off (IBO) of the PA, which can be calculated as following [16].
where $A_i$ is the saturation level of the amplifier input and $P_{in}$ is the average input power of the amplifier.

The investigation on the constellation ring ratio of the 16-APSK (4+12 APSK) modulation scheme, used in the LTE uplink system with the nonlinear distortion due to a PA, is based on the computer simulations. The results are shown in the next section.

### IV. Simulation Results

The simulation of a general SC-FDMA transmission is set up to meet the LTE standard for 5 MHz bandwidth: the IDFT size $M$ of 512, DFT size $N$ of 300, and a cyclic prefix (CP) of 144 [3], [17]. The 16-APSK modulation scheme, with various constellation ring ratios, is used instead of the 16-QAM modulation that is normally used in the LTE standard.

The simulation results include the optimum ring ratio analysis, the Bit Error Rate (BER) analysis, the 16-QAM and 16-APSK performance comparison, the Peak-to-Average Power Ratio (PAPR) analysis, and the 16-QAM and 16-APSK PAPR comparison. The description of the results is shown below.

#### A. Optimum Ring Ratio Analysis

To determine the appropriate range of ratio ($R_2/R_1$) for the 16-APSK modulation scheme in the nonlinear channel, the BER and PAPR performances are shown in Fig. 5 and Fig. 6, with the distortion measured by the IBO of 1 dB and 2 dB, respectively. The BER and PAPR are measured with various SNR. These results show that the ring ratio in the range of 2.5 to 3.5 gives the lowest BER range, but not the highest PAPR values.

Consider more carefully in the above resultant range of 2.5-3.5, the optimum ring ratio for each $Eb/No$ may be estimated by superimposing the BER and PAPR performance curves on the dual-scale data chart. Here, the superimposed chart is chosen because it is commonly used when two data sets have only one axis in common [18].

For example, the superimposed plot of BER and PAPR when $SNR = 5 dB$ and $IBO = 1 dB$, created by the “plotyy()” function in MATLAB, is shown in Fig. 7. The plot indicates that the optimum ring ratio is at $R_2/R_1 = 2.86$ because the BER and the PAPR are optimized (low BER and low PAPR) at their intersection.

<table>
<thead>
<tr>
<th>SNR</th>
<th>IBO 1 dB</th>
<th>IBO 2 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 dB</td>
<td>2.86</td>
<td>2.81</td>
</tr>
<tr>
<td>6 dB</td>
<td>2.75</td>
<td>2.65</td>
</tr>
<tr>
<td>10 dB</td>
<td>2.62</td>
<td>2.60</td>
</tr>
<tr>
<td>15 dB</td>
<td>2.85</td>
<td>-</td>
</tr>
<tr>
<td>20 dB</td>
<td>2.93</td>
<td>2.73</td>
</tr>
</tbody>
</table>
By doing the same to the results getting from the simulation for various SNRs and IBO = 1 dB and 2 dB, the optimum ring ratios are found for each value of SNR as shown in Table I. However, these ratios just give the idea of what the optimum ring ratios should be for the specific values of SNR and IBO. The superimposed plot might provide the optimum ratios differently if the scale of the y-axis is changed. This is the trade-off of the dual-scale chart.

Notice that there is no optimum ratio when SNR = 15 dB and IBO = 2 dB (no intersection of BER and PAPR curves). This might be the scale problem of this plot. Also, in practical implementation, the SNR values could be changed constantly due to the channel condition variation. Therefore, it is impossible to determine the optimum ring ratio specifically and it is more appropriate to state it in range.

### B. Bit Error Rate Analysis

From Fig. 5 and Fig. 6, the ratio R2/R1 gives low PAPR but high BER when the ratio R2/R1 is lower than 2.5. Fig. 8 and Fig. 9 show the simulation results of the system with the IBO of 1 dB and 2 dB, and various values of ring ratios (1.1, 2, 2.5, 2.75, 3, and 3.5), respectively.

In this analysis, we consider at SNR = 20 dB, which gives the highest throughput for the LTE system with four bits per symbol modulation [3]. The BERs from Fig. 8 are 7.5x10^-2, 5x10^-2, 2.5x10^-3, 2x10^-3, 1.6x10^-3, and 1.1x10^-3 for each ring ratio. At the same SNR of 20 dB, the BERs for each ratio of the system with IBO = 2 dB shown in Fig. 9 are 5x10^-2, 2.5x10^-4, 8x10^-5, 5x10^-5, 4.5x10^-5, and 2.5x10^-5. The results in both figures show that the ratio range of 2.5-3.5 gives better BER performance than that of 1.1 and 2. Also, These results agree that the best BER performance occurs when the ratio R2/R1 is equal to 3.5 at high Eb/No.

In addition, the error floor occurs at high SNR and it is caused by the channel nonlinearity based upon the simulation results.

### C. 16-QAM and 16-APSK Performance Comparison

Fig. 10 shows the BER performance comparison of the standard 16-QAM and the 16-APSK with the ring ratio R2/R1 of 3.5. To make a fair comparison, both modulation schemes are set to have the same average energy per symbol. The 16-QAM modulation scheme gives better BER performance than the 16-APSK in the linear channel. But, the 16-APSK modulation scheme performs better than the 16-QAM in the nonlinear channel for LTE system. These outcomes confirm the results in [4]. The error floors associated with the average bit error probability shift down from 5x10^-3 to 6x10^-4 and from 2x10^-4 to 6x10^-5 for the IBO of 1 dB and 2 dB, respectively.

### D. Peak-to-Average Power Ratio (PAPR) Analysis

The Peak-to-Average Power Ratio (PAPR) is analyzed by using Complementary Cumulative Distribution Function (CCDF). The CCDF of PAPR, which is the probability that PAPR is higher than a certain PAPR value PAPR_0 (Pr(PAPR > PAPR_0)), for N subcarriers of an OFDM multi-carrier signal can be calculated by [3]

\[
Pr(PAPR > PAPR_0) = 1 - \left(1 - e^{-PAPR_0}\right)^N
\]

(9)
where $N$ is the number of subcarriers
$PAPR_0$ is the clipping level in the nonlinear system.

The CCDF is measured by using the Monte Carlo simulation for the system applying the 16-APSK (4+12 APSK) modulation with various values of ring ratios in the nonlinear system with IBO of 1 dB and 2 dB. The comparison of the CCDF of PAPR for the ring ratios of 1.1, 2, 2.5, 2.75, 3, 3.5, 4, and 7 is shown in Fig. 11 and Fig. 12.

Consider at the 99 percentile that the $\Pr(PAPR > PAPR_0)$ is less than $10^{-2}$, the ring ratio of 1.1 gives the smallest PAPR while the PAPRs of the others are similar.

To choose the optimum ratio for the 16-APSK, one should consider both the BER and PAPR performances. Then, the ratio of 1.1 is not the appropriate choice for the system even if it has the smallest PAPR because it gives the poorest BER to the system as shown in Subsection B.

### E. 16-QAM and 16-APSK PAPR Comparison

To make a comparison of the system applying the standard 16-QAM and 16-APSK with constellation pattern of 4+12 and ring ratio of 3.5, the average energy per symbol is set to be the same for both modulation schemes. Fig. 13 shows the PAPR performance comparison in terms of CCDF measurement. The system that applies the 16-QAM modulation scheme has a higher PAPR than the 16-APSK in both linear and nonlinear channels for LTE system.

Consider at the 99 percentile that the $\Pr(PAPR > PAPR_0)$ is less than $10^{-2}$, the $PAPR_0$ of the 16-QAM system is equal to 7.6 dB while it is 7.4 dB for the 16-APSK system. This means that the system with 16-APSK outperforms the one with 16-QAM for 0.2 dB or 2.63 percent.

### V. CONCLUSION

LTE technology is attractive in the telecommunications industry due to the high speed data transfer rate it offers. Therefore, the modulation techniques must be selected properly as an important consideration because it can push the system to its fullest potential. The APSK modulation works better in terms of both BER and PAPR performances than QAM in the nonlinear system. So, the APSK modulation is an appropriate modulation scheme to be adopted, as studies have noted in [4], and the ratio of $R2/R1$ is an important factor affecting the PAPR and BER of the APSK. The PAPR increases as the ratio of $R2/R1$ increases. While BER changes differently to the $R2/R1$ ratio and is shown to have one range with a low BER. In this paper, it is shown that the optimal $R2/R1$ ratio of the 4+12 APSK is more suitable to state in range, which is between 2.5 to 3.5. This optimal ratio range gives the best results in terms of the low BER and PAPR to the nonlinear LTE uplink system. Also, the 16-APSK with the proposed ring ratio provides better BER and PAPR performances compared with the standard system with 16-QAM in the nonlinear system. The authors hope that this paper will help to drive the performance of the LTE system that makes the implementation widespread in the future.
REFERENCES


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