A Novel QC-LDPC Code with Flexible Construction and Low Error Floor

Hanxin WANG[1,2], Shaoping CHEN[1,2], Cuitao ZHU[1,2] and Kaiyou SU[1]

[1]*Department of Electronics and Information Engineering, South-Central University for Nationalities, Wuhan, China*

[2]*Hubei key Laboratory of Intelligent Wireless Communications, Wuhan, China*

wanghx8888@163.com, spchen@scuec.edu.cn, zhucuitao@163.com, sukaiyou@gmail.com

***Abstract*-- Slide rectangular window structure for QC-LDPC codes (SRW-QC-LDPC) with flexible code lengths and code rates is proposed, which aim to eliminate the cycles of length 4 without computer search. The parity-check matrix would have different extension factors and structures by using the slide rectangular window in the base matrix, the degree distribution is optimized by the optimal diagonal method. Because the dual-diagonal structure with many variable nodes of degree-2 may lead to high error floor, SRW-QC-LDPC codes with quasi tri-diagonal structure are also proposed by changing the location of the third diagonal to partly eliminate variable nodes of degree-2 for lower error floor. Simulation results show that SRW-QC-LDPC codes with quasi tri-diagonal structure can not only flexibly expand the code lengths and code rates but also reduce the encoding complexity and improve the BER performance compared to quasi dual-diagonal structure in IEEE802.16e QC-LDPC codes. The novel QC-LDPC codes are available and suitable for the adaptive transmission systems and hardware implementation.**

***Keywords*— QC-LDPC codes; Diagonal Structure; Degree Distribution; Encoding Complexity; Error Floor**

I. INTRODUCTION

QC-LDPC codes are quasi cyclic low density parity check codes with simple structure and efficient encoding algorithm, they are characterized by the parity-check matrix H which consists of small square blocks which are the zero matrix or circulant permutation matrices. QC-LDPC codes have been widely used in the fields of mobile communication, such as IEEE802.16e, DVB-T2 and IEEE802.11n, etc.

In the construction of LDPC code's matrix H, it should be considered to avoid the existence of cycles of length 4. Cycle refers to the number of edges which form a closed loop in the corresponding Tanner graph of matrix H, the minimum length of cycles is called girth[1] [2]. By optimizing matrix H, the larger girth can be realized to improve error correction capability of QC-LDPC codes. BIBD algorithm is proposed to construct the girth 10 for matrix H in [3]. The girth of matrix H is increased to 18 by constructing the mother matrix in [4]. Although the large girth is helpful to obtain better performance, only a few LDPC codes meet the requirements, thus the practical application of LDPC codes is limited. No cycles of length 4 is essential to optimize the number of cyclic shifting and degree distribution of LDPC codes to improve their performance. The typical schemes include IEEE802.16e QC-LDPC codes with girth 6, the maximized ACE and minimized BER criterion algorithm, but the complexity of these algorithms is higher. In addition, the arithmetic progression method can also rapidly construct the matrix H without cycles of length 4, but the BER performance is not good enough [5] [6] [7] [8]. When the base matrix is larger, the length of the permutation matrix is also very long, the range of code length is limited so as to not be suitable for adaptive transmission system with the varied code lengths and code rates [9]. QC-LDPC codes usually adopt lower triangular structure of quasi dual-diagonal, coding efficiency and BER performance is not good [10]. In order to achieve more efficient encoding, several improved structures have been put forward, such as tri-diagonal structure, new lower triangle structure and backward iteration structure [11] [12] [13] [14]. However, tri-diagonal structure is not flexible, the new lower triangle structure and backward iteration structure contain many variable nodes of degree-2 similar to the quasi dual-diagonal structure, which will lead to the higher error floor. The above improved QC-LDPC codes are mostly restricted in the code lengths and code rates flexibility. This paper firstly presents a novel slide rectangular window (SRW) scheme based on the arithmetic progression to construct matrix H of QC-LDPC codes and uses the optimal diagonal method to optimize the degree distribution of matrix H. Secondly, a kind of quasi tri-diagonal structure is proposed, it can partly eliminate the variable nodes of degree-2 by adjusting the location of the third diagonal to achieve lower error floor. The experimental results show that the proposed SRW-QC-LDPC codes with quasi tri-diagonal structure can not only flexibly expand the code lengths and code rates but also reduce the encoding complexity and improve the BER performance compared to quasi dual-diagonal structure in IEEE802.16e QC-LDPC codes. The novel QC-LDPC codes are available and suitable for the adaptive transmission systems and hardware implementation.

The rest of this paper is organized as follows. In section 2, we present the SRW scheme to construct QC-LDPC codes and use the optimal diagonal method to optimize the degree distribution. In section 3, the proposed quasi tri-diagonal structure and fast encoding algorithm is introduced. The simulation results for SRW-QC-LDPC codes with the quasi tri-diagonal structure are shown in section 4 and the conclusions are drawn in the last section.

II. QC-LDPC CODES WITH SLIDE RECTANGULAR WINDOW STRUCTURE

A. Sliding rectangular window construction for matrix H_{b1}

The base matrix H_b for parity-check matrix H of LDPC codes are composed of a group of zero matrix and circulant permutation matrices, it is shown as follows:

$$H_b = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n_b} \\ S_{21} & S_{22} & \cdots & S_{2n_b} \\ \vdots & \vdots & \ddots & \vdots \\ S_{m_b 1} & S_{m_b 2} & \cdots & S_{m_b n_b} \end{bmatrix} \quad (1)$$

where $S_{ij}(1 \le i \le m_b, 1 \le j \le n_b) \in (-1, 0, N)$ is the circulant permutation matrices in ith row and jth column of the matrix H_b, -1 denotes zero matrix, 0 denotes identity matrix, the positive integer N denotes right cyclic shifting matrix from identity matrix. If the expansion factor z denotes dimension of permutation matrix, the matrix H_b will be extended to obtain matrix H with the size of $(z*m_b)*(z*n_b)$.

In the construction of the matrix H_b of QC-LDPC codes, it is important to avoid the existence of the cycles of length 4. In order to realize no cycles of length 4, z satisfies a certain value and S_{ij} must satisfy:

$$S_{ij} = \begin{cases} i & j = 1, 1 \le i \le m_b \\ S_{i(j-1)} & 2 \le j \le n_b, 1 \le i \le m_b \end{cases} \quad (2)$$

The base matrix H_b of QC-LDPC codes based on the quasi dual-diagonal structure can be expressed as follows:

$$H_b = \begin{bmatrix} H_{b1} & H_{b2} \end{bmatrix} \quad (3)$$

$$H_{b1} = \begin{bmatrix} \overbrace{\begin{matrix} h_{11} & h_{12} & \cdots & \cdots & h_{1k_b} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ h_{r_b 1} & h_{r_b 2} & \ddots & \ddots & h_{r_b k_b} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ h_{m_b 1} & h_{m_b 2} & \cdots & \cdots & h_{m_b k_b} \end{matrix}}^{H_{b1}} \end{bmatrix} \quad (4)$$

$$H_{b2} = \begin{bmatrix} \overbrace{\begin{matrix} b(1) & 0 & -1 & \cdots & -1 \\ \vdots & 0 & 0 & \ddots & \vdots \\ b(r_b) & -1 & \ddots & \ddots & -1 \\ \vdots & \vdots & \ddots & 0 & 0 \\ b(m_b) & -1 & \cdots & -1 & 0 \end{matrix}}^{H_{b2}} \end{bmatrix} \quad (5)$$

where H_{b1} and H_{b2} is the information bits and the parity bits portion of matrix H_b, respectively. H_{b1} with the size of m_b*k_b was sparse matrix without cycles of length 4. H_{b2} with the size of m_b*m_b is a fixed structure matrix, 0 and -1 denotes the identity matrix and zero matrix, respectively, $b(1)=b(m_b)$ is the prime number less than z, $b(r_b)=0$.

According to (2), matrix H_{b1} without cycles of length 4 is constructed as follows:

$$h_{ij} = \begin{cases} hf + (i-1) \cdot rt & j = 1 \\ h_{i1} + (i-1) \cdot (ct + (j-1)) & 1 < j \le k_b \end{cases} \quad (6)$$

The corresponding z should satisfy:

$$z \ge (m_b - 1) \cdot rt + (k_b - 1) \cdot (ct + m_b - 1) \quad (7)$$

where $1 \le i \le m_b$, hf is elements value of the first row and first column in the matrix; ct and rt is the common difference of the first row and first column, respectively; m_b and k_b is the row and column number of the matrix H_{b1}, respectively.

Due to the code rate $R=k_b/(k_b+m_b)$, the code rates and code lengths can flexibly change by adjusting m_b and k_b value. For example, $m_b=k_b=6$, $R=1/2$, the minimum code length is 6*62=372, step length is 12; If R want to 1/3, one way is to put $k_b=3$ while m_b maintain, it is equivalent to delete three columns in the matrix H_{b1}, the corresponding minimum code length becomes 9*16=144, step length 9; Another way is to put $m_b=4$ and $k_b=2$, it is equivalent to delete two rows and four columns of the matrix H_{b1}, the minimum code length turn into 6*7=42, step length into 6. Obviously, the code rates and code lengths transformation is very flexible without matrix reconstruction but simple processing of row and column. In addition, the different matrix H_{b1} and z can be constructed by modifying the value of hf, ct and rt. For instance, while $hf=1$, $ct=0$ and $rt=1$, a matrix with the size of 6*6 is constructed in the solid rectangular window and when hf, ct and rt value is 3, 1 and 2, the matrix will become the dashed rectangular window as shown in Fig.1. It seems to slide rectangular window to the right and low position in matrix, thus named as Slide Rectangular Window (SRW) method.

The specific steps of matrix H_{b1} construction with SRW method are as follows:

Step 1: On the basis of (6), determine a rectangular window matrix, the size is m_b*k_b.

Step 2: According to (7), determine the corresponding extension factor z and slide rectangular window's position in the matrix.

Step 3: Regard slide rectangular window matrix as the matrix H_{b1}, and combine it with matrix H_{b2} whose size is m_b*m_b, then matrix H_b is finished.

To construct matrix H_{b1} with SRW, the corresponding extension factor z is smaller, it can gain flexible and diverse code lengths and code rates of QC-LDPC codes, it is more suitable for adaptive transmission system.

$$H_{b1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 5 & 7 & 9 & 11 & 13 & 15 & 17 & 19 \\ 4 & 7 & 10 & 13 & 16 & 19 & 22 & 25 & 28 \\ 5 & 9 & 13 & 17 & 21 & 25 & 29 & 33 & 37 \\ 6 & 11 & 16 & 21 & 26 & 31 & 36 & 41 & 46 \\ 7 & 13 & 19 & 25 & 31 & 37 & 43 & 49 & 55 \\ 8 & 15 & 22 & 29 & 36 & 43 & 50 & 57 & 64 \\ 9 & 17 & 25 & 33 & 41 & 49 & 57 & 65 & 73 \end{bmatrix}$$

Figure 1. Slide rectangular window

B. Optimal diagonal method for Matrix H_{b1}

In SRW method, regards directly slide rectangular window matrix as H_{b1} and then constructs H_b, the degree of rows and columns is the same. It is equivalent to a regular LDPC code whose BER performance is poor [11]. We adopt the optimal diagonal method to modify the matrix H_{b1} and make it have the different degree distribution to improve BER performance of

LDPC codes. Specific means is to use -1 instead of certain diagonal elements in the matrix H_{b1} with SRW, i.e. using zero matrix instead of the permutation matrix, it will realize the optimal degree distribution. E.g. the solid rectangular window matrix in Fig.1 is modified to get the matrix $\tilde{H}_{b1}$ as follows:

$$\tilde{H}_{b1}=\begin{bmatrix} -1 & 1 & -1 & 1 & 1 & 1 \\ 2 & -1 & 4 & -1 & 6 & 7 \\ -1 & 5 & -1 & 9 & 11 & 13 \\ 4 & -1 & 10 & -1 & 16 & 19 \\ -1 & 9 & -1 & 17 & 21 & 25 \\ 6 & -1 & 16 & -1 & 26 & 31 \end{bmatrix} \tag{8}$$

The optimized matrix $\tilde{H}_{b1}$ combined the matrix H_{b2} to obtain matrix H_b as follows:.

$$H_b=\begin{bmatrix} -1 & 1 & -1 & 1 & 1 & 1 & 5 & 0 & -1 & -1 & -1 & -1 \\ 2 & -1 & 4 & -1 & 6 & 7 & -1 & 0 & 0 & -1 & -1 & -1 \\ -1 & 5 & -1 & 9 & 11 & 13 & 0 & -1 & 0 & 0 & -1 & -1 \\ 4 & -1 & 10 & -1 & 16 & 19 & -1 & -1 & -1 & 0 & 0 & -1 \\ -1 & 9 & -1 & 17 & 21 & 25 & -1 & -1 & -1 & -1 & 0 & 0 \\ 6 & -1 & 16 & -1 & 26 & 31 & 5 & -1 & -1 & -1 & -1 & 0 \end{bmatrix} \tag{9}$$

After optimal diagonal method for matrix H_b with SRW, the BER performance of the constructed QC-LDPC codes is more improved. Meanwhile, the code lengths and code rates are flexible and various, it is adaptive in wireless mobile communication system.

III. QC-LDPC CODES WITH QUASI TRI-DIAGONAL STRUCTURE

A. Quasi tri-diagonal construction for matrix H_{b2}

QC-LDPC codes mostly use the quasi dual-diagonal structure for matrix H_{b2}. In order to improve coding efficiency and BER performance, we present a quasi tri-diagonal structure for matrix H_{b2} in this section. The existing optimal structures for matrix H_{b2} include tri-diagonal structure, new lower tri-triangle structure and reverse iterative structure. In the tri-diagonal structure, three diagonal elements are not -1, one of them is zero, the other two are natural numbers and their values need to meet certain condition, so the code lengths and code rates are restricted. In new lower tri-triangle structure, three elements in the first column is not -1 and two elements in the rest column is not -1. In reverse iterative structure, the elements of two diagonal are 0, the rest are 1. The variable nodes of degree-2 accounts for the vast majority and these low degree variable nodes will lead to the high error floor. To solve the disadvantages of these existing structures, we propose a quasi tri-diagonal construction for matrix $\tilde{H}_{b2}$ as follows:

$$\tilde{H}_{b2}=\begin{bmatrix} 0 & & & & & \\ 0 & 0 & & & -1 & \\ \vdots & 0 & \ddots & & & \\ r(1) & & \ddots & \ddots & & \\ \vdots & \ddots & & \ddots & 0 & \\ -1 & \cdots & r(l) & \cdots & 0 & 0 \end{bmatrix} \tag{10}$$

Where, the element of matrix $\tilde{H}_{b2}$ is only 0 or -1, and only three diagonal elements are 0, $r(i)\in 0$, $r(1)$ is the start position of the third diagonal, r (l) is the end position. In order to satisfy the no cycles of length 4, the row values of r (*1*) should not be less than 3.

From (10), it can be seen that the quasi tri-diagonal structure has the advantages of flexibly adjusting the third diagonal position compared with tri-diagonal structure. And the quasi tri-diagonal structure can also partly eliminate the variable nodes of the degree-2 compared to the new lower tri-triangle and reverse iterative structure. With the increase of $\tilde{H}_{b2}$ matrix dimension, the selected third diagonal will be various, the degree distribution is also different, so we can properly select the third diagonal position to optimize the degree distribution so as to improve BER performance of the QC-LDPC codes.

Using SRW method in section II, the matrix H_{b1} is constructed at m_b=9 and k_b=9. According to (10), the matrix H_{b2} is constructed. Two matrices are combined to the matrix H_b as shown in figure 2. It can be seen that the third diagonals have 6 kinds of selection which uses oblique line to label the third diagonal position. When one of the labeled third diagonals is selected, then the elements -1 on this diagonal is replaced by 0.

$$H_b=\begin{bmatrix} -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 2 & -1 & 4 & -1 & 6 & 7 & -1 & 9 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & 5 & -1 & 9 & -1 & 13 & 15 & -1 & 19 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 \\ 4 & -1 & 10 & -1 & 16 & -1 & 22 & 25 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & -1 \\ -1 & 9 & -1 & 17 & -1 & 25 & -1 & 33 & 37 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & -1 \\ -1 & -1 & 16 & -1 & 26 & -1 & 36 & 41 & 46 & -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 \\ 7 & -1 & -1 & 25 & -1 & 37 & -1 & 49 & 55 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 \\ -1 & 15 & -1 & -1 & 36 & -1 & 50 & -1 & 64 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & -1 \\ 9 & -1 & 25 & -1 & -1 & 49 & -1 & 65 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 \end{bmatrix}$$

Figure 2. Matrix H_b with SRW and quasi tri-diagonal structure

B. Encoding algorithm for quasi tri-diagonal structure

The specific steps of fast encoding algorithm for the quasi tri-diagonal structure are as follows:

Step 1: Information bits s and parity check bits p is segmented, each segment length is z, the codeword c is expressed as follows:

$$c=\left[s \mid p\right]=\left[\begin{matrix} s_1 & s_2 & \cdots & s_{k_b} \mid p_1 & p_2 & \cdots & p_{m_b} \end{matrix}\right] \tag{11}$$

where k_b=k/z, m_b=m/z.

Step 2: According to the parity-check matrix equation $H\bullet c^T=0$, p can be calculated by:

$$p_1=\sum_{j=1}^{k_b} Z_{b1}(1,j)\bullet s_j \tag{12}$$

$$p_i=\begin{cases} p_{i-1}+\sum_{j=1}^{k_b} Z_{b1}(i,j)\bullet s_j, & i=2,\cdots,c_r-1 \\ p_{i-c_r+1}+p_{i-1}+\sum_{j=1}^{k_b} Z_{b1}(r,j)\bullet s_j, & i=c_r,\cdots,m_b \end{cases} \tag{13}$$

where c_r is the row value of r(1) in matrix $\tilde{H}_{b2}$ and $Z_{b1}(i,j)$ is the permutation matrices of ith row and jth column in the matrix $\tilde{H}_{b1}$.

Step 3: Combine $p=\left[\begin{matrix} p_1 & p_2 & \cdots & p_{m_b}\end{matrix}\right]$ from step 2 and information bits s to end up with codeword c.

In the quasi dual-diagonal structure, p_1 is calculated by:

$$p_1 = \left(Z_1 + Z_r + Z_{m_b}\right)^{-1} \bullet \sum_{i=1}^{m_b}\sum_{j=1}^{k_b} Z_{b1}(i,j) \bullet s_j \quad (14)$$

Obviously, the calculation amount of p_1 in (12) is smaller than that in (14). That is to say, the computational complexity of the quasi tri-diagonal structure coding algorithm is lower than the quasi dual-diagonal. Tab.1 shows the comparing results for $p1$ computation of quasi tri-diagonal and quasi dual-diagonal structure. Here, the code rate is 1/2, code length of n, extension factor is z.

Table 1. Comparison for p_1 computation of two structures

Diagonal structure	Addition operation	Multiplication operation
quasi-dual	$(z\text{-}1)*n/(4z)$	$n^2/4$
quasi-triple	$(z\text{-}1)*n/2$	$nz/2$

IV. SIMULATION RESULTS AND ANALYSIS

A. Simulation results of SRW-QC-LDPC codes

The simulation experiments are under the fast iterative coding, BPSK modulation, LLR BP decoding, 20 iterative number and AWGN channel.

In Tab.2, m_b = 6 and k_b takes different values, then the code rates change from 1/7 to 5/8, and the code lengths also vary from 42 to 816 with the different steps. It is shown that the code lengths and code rates for SRW-QC-LDPC codes are flexible, it can meet the need of adaptive transmission system.

Table 2. Parameters of SRW-QC-LDPC

m_b	k_b	Code Rate	Code length (t=0,1,2,...)
6	1	1/7	42+7t
6	2	1/4	88+8t
6	3	1/3	144+9t
6	4	2/5	210+10t
6	5	5/11	286+11t
6	6	1/2	372+12t
6	7	7/13	458+13t
6	8	4/7	574+14t
6	9	3/5	690+15t
6	10	5/8	816+16t

Fig.3 gives the BER performance comparison of basic check matrix with the different degrees before and after the optimal diagonal method. We can find that the BER performance is a little improved when the degree is less than 6, but the optimal diagonal method has obvious advantages when the degree is greater than 6 and SRW-QC-LDPC codes BER performance is greater improved.

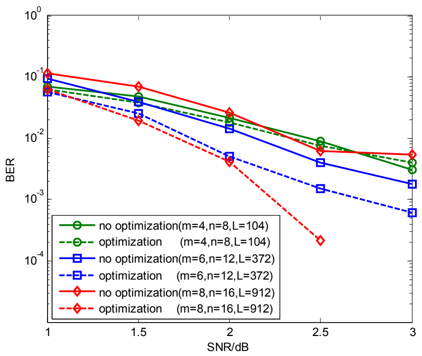

Figure 3. BER with the optimal diagonal method

In IEEE802.16e QC-LDPC codes, there are many code lengths and code rates, so we choose 4 code lengths 576, 960, 1440 and 2304 to do simulations. The experimental results for SRW-QC-LDPC and 802.16e-QC-LDPC codes are shown in Fig.4.

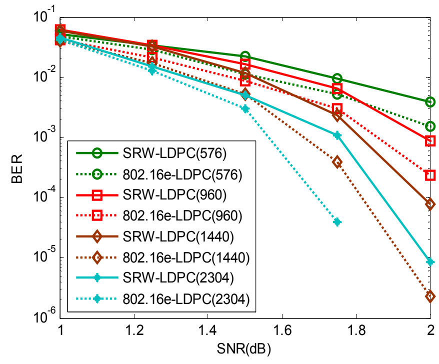

Figure 4. BER for SRW-QC-LDPC and 802.16e-QC-LDPC

It can be seen from Fig.4 that SRW-QC-LDPC codes are slightly losses on the BER performance compared to 802.16e QC-LDPC codes, but the signal-to-noise ratio (SNR) is more than 2dB, the BER of SRW-QC-LDPC codes with the code length 2304 can reach 10^{-5}, it can satisfy the demand of system.

Tab.3 shows the parameters of SRW-QC-LDPC codes compared with 802.16e standard and literature [12]. It can be seen that the code rates of 802.16e-QC-LDPC codes have only four of 1/2, 2/3, 3/4 and 5/6, the code lengths have a total of 19 from the minimum 576 to maximum 2304 which the step length is 96. In literature [12], the code rates are expanded to a total of 10 from 1/4 to 3/4, the minimum code length is 336, the step length is on a total of 6 from 4 to 9. The code rates of SRW-QC-LDPC codes can expand without limits, the minimum code length reaches to 42, the minimum step length is 4. It obviously indicates that SRW-QC-LDPC codes can flexibly adjust the code rates and the code lengths to be more available and suitable for adaptive transmission system.

Table 3. Parameters of QC-LDPC with different matrices H

	IEEE802.16e	Reference [12]	SRW
Code rates	1/2,2/3,3/4,5/6	1/4,...,2/3,...,3/4	unlimited
Code lengths	576-2304	336, ...	42, ...
Step size	96(t+1),(t=0,1,... ...)	4,5,6,7,8,9	4, ...

B. Simulation results and analysis of SRW-QC-LDPC codes with difference quasi tri-diagonal structure

The parity check matrix H can be obtained from the base matrix H_b in Fig.2. By selecting the different third diagonal position of matrix H_b, the BER performance is simulated for SRW-QC-LDPC codes with difference quasi tri-diagonal. The simulation results are shown in Fig.5, where frame number is 300, the code length 1152, the code rate 1/2, extension factor z=64. It can be observed from Fig.5 that the BER performance in the Ⅱ and Ⅲ diagonal is significantly better than the others, and the Ⅵ diagonal is worst. It implies that the third diagonal of matrix H_b selects Ⅲ position is the best in AWGN channel.

Due to the different degree distributions in the different channels, SRW-QC-LDPC codes with different quasi tri-diagonal have more advantages because of their flexible and optimal degree distributions.

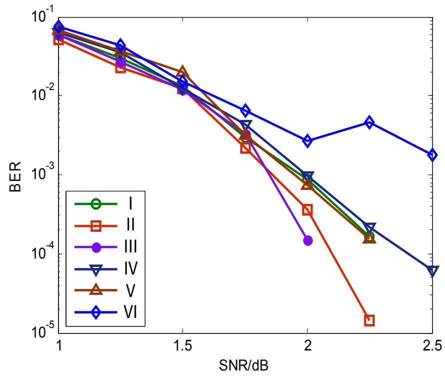

Figure 5. BER for SRW-QC-LDPC with different quasi tri-diagonal

Fig.6 gives the BER performance for QC-LDPC codes under the same matrix H_{b1} and the different matrix H_{b2} which is the quasi dual-diagonal and quasi tri-diagonal structure, respectively. Here, the code lengths take 576, 960 and 1152, the frame number is 1000, 500 and 300, the code rate is 1/2, the matrix H_b of quasi tri-diagonal structure is taken the size of the (6, 12), (8, 16) and (9, 18).

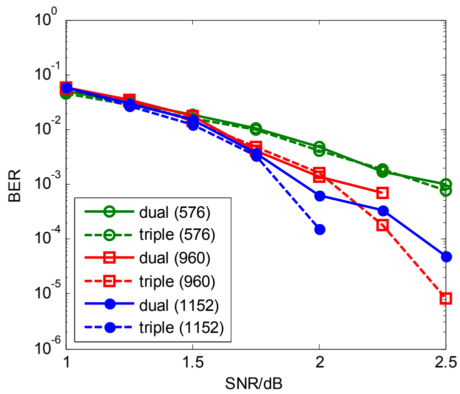

Figure 6. BER comparison for QC-LDPC with quasi dual-diagonal and quasi tri-diagonal structure

It is obvious in Fig.6 that the BER performance of quasi tri-diagonal structure has more and more obvious advantage with the increase of the code lengths. E.g. When BER reaches 10^{-4}, the SNR of quasi dual-diagonal structure needs 2.4dB, but the SNR of quasi tri-diagonal only needs 2.05 dB, the coding gain is nearly 0.35dB. This is because with the increase of the code length, the third diagonal choice for the quasi tri-diagonal structure gets more, the degree distribution is more flexible variation, therefore the chance to near a good degree distribution is greater.

Fig.7 gives the BER performance comparison between the quasi tri-diagonal SRW-QC-LDPC codes and 802.16e QC-LDPC codes under the different code lengths. Here, the code lengths take 576, 960 and 1152, the frame number is 1000, 500 and 300 respectively, the code rate is 1/2, the matrix H_b of quasi tri-diagonal structure is taken the size of the (6, 12), (8, 16) and (9, 18).

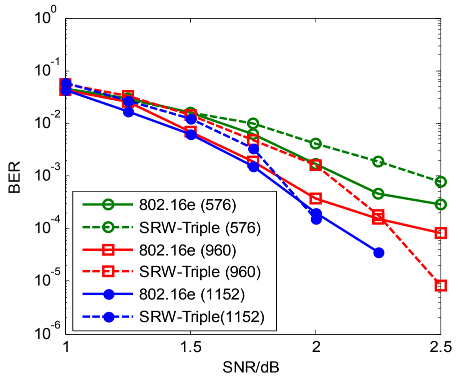

Figure 7. BER for 802.16e QC-LDPC and quasi tri-diagonal SRW-QC-LDPC

It is clear in Fig.7 that the BER performance of quasi tri-diagonal SRW-QC-LDPC codes is poorer when the code length is 576. But the BER performance of quasi tri-diagonal SRW-QC-LDPC codes is near to IEEE802.16e LDPC codes when the code length is 960 and 1152. In addition, the BER performance of quasi tri-diagonal SRW-QC-LDPC codes is even better than the IEEE802.16e LDPC codes at the higher SNR and longer code lengths.

V. CONCLUSIONS

This paper proposed a novel SRW-QC-LDPC code with quasi tri-diagonal structure which can flexibly change the code rates and code lengths with lower complexity, it is more available and suitable for adaptive transmission system. The simulation results show that the presented structure can not only improve the BER performance of QC-LDPC codes but also reduce the encoding complexity compared with dual-diagonal structure. When the code length is the longer, the BER performance of SRW-QC-LDPC codes with quasi tri-diagonal construction is near to IEEE802.16e LDPC codes and is even better than the 802.16e standard at the higher SNR.

ACKNOWLEDGMENT

This work was sponsored by the National Natural Science Foundation of China with Grant No.61072075 and the Natural Science Foundation of Hubei Province with Grant No.2013CFB448, it was also supported by the Key Project of the Fundamental Research Funds for the Central Universities, South-Central University for Nationalities with Grant No.CZZ13001.

REFERENCES

[1]Fossorier M P. Quasi-Cyclic Low Density Parity Check Codes From Circulant Permutation Matrices[J]. *IEEE Trans. Info. Theory*, 2004, 50(8): 1788-1793.

[2] Tanner R M. A recursive approach to low complexity codes [J]. *IEEE Trans. Info. Theory*, 1981, 27(5): 533-548.

[3] Zhang Fan, Mao Xuehong, Zhou Wuyang.Girth-10 LDPC codes based on 3-D cyclic lattices [J]. *IEEE Trans. on Vehicular Technology*, 2008, 57(2): 1049-1060.

[4] Kim S, No J and Chung H. On the girth of Tanner (3,5) Quasi-Cyclic LDPC codes [J]. *IEEE Trans. Info. Theory*, 2006, 52(4): 1739-1744.

[5] IEEE 802.16e D5 Amendment to IEEE Standard for Local and Metropolitan Area Networks, Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems[S].
[6] Kang J, Fan P, Cao Z. Flexible construction of irregular partitioned LDPC codes with low error floors [J]. *IEEE Communications Letters*, 2005, 9(6): 534-536.
[7] Myung S, Yang K. Lifting methods for quasi-cyclic LDPC codes [J]. *IEEE Communications Letters*, 2006, 10(6): 489-491.
[8] Sharon E, Lisyn S. Constructing LDPC codes by error minimization progressive edge growth [J]. *IEEE Trans. Commun.*, 2008, 56(3): 359-368.
[9] Peng Li, Zhu Guangxi. Cyclic shift number design of permutation matrix for QC-LDPC codes [J]. *Chinese Journal of Electronics*, 2010, 38(4): 786-790.
[10] Luby M G, Mitzenmacher M, Shokrollah M A. Improved Low-density Parity-check Codes Using Irregular Graphs[J]. *IEEE Trans. Info. Theory*, 2001, 47(2): 585-598
[11] Y. Xu and G. Wei. On the Construction of Quasi-Systematic Block-Circulant LDPC codes [J]. *IEEE Commun., Letters*, 2007, 11(11): 886-888.
[12] Wai M. Tam, Francis C. M. Lau and Chi K. Tse. A Class of QC-LDPC Codes with Low Encoding Complexity and Good Error Performance [J]. *IEEE Commun. Letters*, 2010, 14(2): 169-171.
[13] Guo Rui, Hu Fangning, Liu Jilin. QC-LDPC codes construction method with low error floor and linear complexity [J]. *Chinese Journal of Circuits and System*, 2011, 16(6): 87-93.
[14] Yige Wang, Draper S.C and Yedidia,J.S. Hierarchical and High Girth QC LDPC Codes[J]. *IEEE Trans. Info. Theory*, 2013, 59(7): 4553-4583.