Source Nodes Power Optimization in Energy Harvesting Two-Way Relay Networks

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***Abstract*—In this paper, we investigate the outage performance for the two-way relaying model in which a half-duplex energy harvesting relay assists in the bi-directional communication of two source nodes. Closed form results for the outage probability of two-way relaying networks has been introduced and corrected in the scenario of an energy harvesting relay. We then consider the relationship between source nodes power allocation, relay energy harvesting efficiency and outage probability. The two source nodes share the limited total power, to minimize the outage probability, we derive closed form results for the source nodes power allocation with fixed relay energy harvesting efficiency. Simulation results are provided to confirm the analytical results.**

Keywords—Two-way relay, cooperative communication, outage probability, power allocation, energy harvesting.

I. Introduction

In wireless networks, due to the limited capacity of batteries and the difficulty of frequent battery recharging or replacement, energy is a scarce and precious resource. Therefore a technique to collect energy from the surrounding environment, called energy harvesting (EH) [1], has become an appealing solution to overcome the bottleneck of energy constrained wireless networks [2]. The authors introduced a new concept of energy harvesting which involves collecting energy from ambient radio frequency signals in [3]. In [4], simultaneous information and power transfer over the wireless channels has been studied. The fundamental tradeoff between transmitting energy and transmitting information over a single noisy line has been studied [5].

Cooperating between multiple terminals [6] can effectively improve the wireless coverage and the resistance to channel fading. However, due to the half-duplex in practical communication systems, cooperative relaying cause damage to the spectrum effectiveness. Two-way relaying based on AF (Amplify-an-Forward) and DF (Decode-and-Forward) agreement for the typical three point networks was proposed in [7]. Higher throughput of system can be achieved by two-way relaying in [8]. However, outage happens at any point will make the transmission impossible in two-way relaying systems. In [9], the closed form expressions of outage probability in two-way relaying is derived.

Thus, this two technology, energy harvesting and two-way relaying, have attracted great interest. In this paper, we consider the typical three point two-way relaying system, but the relay here will use energy harvesting. The system model is shown in fig. 1. The source nodes A and B share the limited total power, the two-way relay R uses energy harvesting. The energy harvesting efficiency is a parameter which means the rate of the energy that harvested from the signal. We first study the outage probability of this system, and give the closed form results with power allocation and energy harvesting efficiency to minimize the outage probability. To minimize the outage probability, we derive closed form results for the optimal source nodes power allocation with fixed relay energy harvesting efficiency for this two-way relay networks. Simulation results are given to confirm the analytical results. Cases of different energy harvesting efficiencies are considered and compared with each other.

The rest of the paper is organized as follows. In Section II, we describe the system model. The outage probability of this system is derived in Section III, and power allocation in this system will be discussed. In Section IV, simulation results are given to confirm the analytical results, cases of different η are discussed. In Section V, we draw the main conclusions.

II. System Model

Fig. 1 shows the system model. A and B are two source nodes, R is the energy harvesting relay in their bi-directional communication. All these terminals operate in half-duplex mode. It is the typical three point model for two-way relaying networks, but the relay here is an energy harvesting relay which has no transmit power itself, and using the energy harvested from signals to transmit. There are three phases in the communication. First, in the transmission phase, two source nodes A and B transmit to the relay R simultaneously. Second, in the processing phase, R harvests energy from the source nodes signals and operates certain signal processing to receive information. Last, in the relaying phase, R broadcasts the received information.

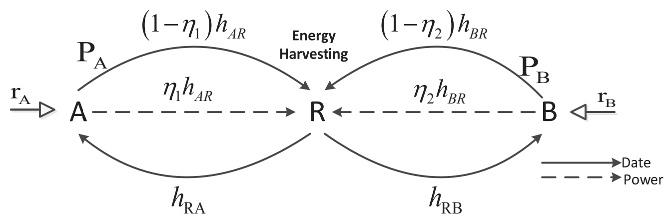

Fig. 1. Two-way relaying model with energy harvesting.

η means the rate of energy harvested from receive power, express as:

$$Q = \eta P. \quad (1)$$

P is the receive power. Q is the harvested power charges from the signals. In this paper, we assume that R uses all the harvested power from A and B to broadcasts the received information in the third phase. Obviously, $0 \leqslant \eta < 1$ is the limit.

Then the else parameters are defined as follow. r_A and r_B are the corresponding data rates at source nodes A and B. P_A and P_B are the source nodes transmit power, and share a limited total power P_T:

$$P_A + P_B = P_T. \quad (2)$$

P_R is the broadcast power in relaying phase, which is the sum of harvested power from A and B. h_{AR} and h_{BR} are the independent channel coefficients for A to R and B to R respectively. Let $R_A = 2^{2r_A} - 1$ and $R_B = 2^{2r_B} - 1$, we define $z = R_B/R_A$, called amendatory coefficient of service type. η_1 and η_2 are energy harvesting efficiency for each terminal. Denoting P_{AI} and P_{BI} as the information power, we have $P_{AI} = (1 - \eta_1)P_A$ and $P_{BI} = (1 - \eta_2)\,P_B$. They mean the part of power that really transmits dates at terminals.

We have some assumes as follow: first, the channels are reciprocal, thus we have $h_{AR} = h_{RA}$ and $h_{BR} = h_{RB}$, then denote $g_1 = |h_{AR}|^2$ and $g_2 = |h_{BR}|^2$. Second, the variance of zero-mean additive white Gaussian noise (AWGN) denoted by σ^2 is assumed to be equal at all terminals. Last, every point knows the exact channel information.

III. OUTAGE PROBABILITY AND POWER ALLOCATION

A. *Outage Probability*

Cooperative relaying can effectively improve the wireless coverage and the resistance to channel fading, though the cooperating between multiple terminals. But, outage happens at any point will make the transmission impossible in two-way relaying systems. So it is important to study the outage probability of relay networks and make solutions to reduce the probability. With the assume we have given in section II, source node A know the information which it sends in first phase perfectly, it can completely remove this interference signal from receive signal. Thus the instantaneous SNR γ_1 and γ_2 can be given [9]:

$$\gamma_1 = \frac{P_B P_R |h_{AR}|^2 |h_{BR}|^2}{\left((P_A + P_R)\,|h_{AR}|^2 + P_B |h_{BR}|^2 \right) \sigma^2}, \quad (3)$$

$$\gamma_2 = \frac{P_A P_R |h_{AR}|^2 |h_{BR}|^2}{\left((P_B + P_R)\,|h_{BR}|^2 + P_A |h_{BR}|^2 \right) \sigma^2}. \quad (4)$$

In the two-way transmission, outage happens at any point will break off the system. So we can get the outage probability as follow [10]:

$$\begin{aligned} P_{out} &= P\left(I_A < r_A \cup I_B < r_B\right) \\ &= P\left(\gamma_1 < R_A \cup \gamma_2 < R_B\right). \end{aligned} \quad (5)$$

From above, we can get the closed form results of the outage probability for the traditional three point model [9]. However in this paper, the relay uses energy harvesting and has no power itself. So P_A and P_B will contribute part of themselves to R for broadcast phase. In other words, the power P_A and P_B transmit in this system take part in both information transmission and energy transmission.

Thus we can divide the whole transmission into two parts: information and energy. The power transmit information will only be $(1-\eta_1)P_A$ and $(1-\eta_2)P_B$, and the other parts $\eta_1 P_A$ and $\eta_2 P_B$ transmit energy to R, shows in fig. 1.

The information transmission is just the same as the typical three point model, and energy transmission has no direct relationship with outage. Then defining $P_{AI} = (1 - \eta_1)P_A$ and $P_{BI} = (1 - \eta_2)\,P_B$, we can get the results.

When $\frac{P_{BI}}{P_{AI}+P_R} \geqslant z$, the outage probability only depends on the link $A \to R \to B$. Thus when the noise power $\sigma^2 \ll P_T$, we can get the approximate result in this case as follow:

$$P_{out1} = 1 - \exp\{-\frac{g_1\,(P_{BI} + P_R) + g_2 P_{AI}}{P_{AI} P_R} R_A \sigma^2\}. \quad (6)$$

When $\frac{P_{AI}}{P_{BI}+P_R} \geqslant \frac{1}{z}$, in a similar way, the outage probability only depends on the link $B \to R \to A$ and when the noise power $\sigma^2 \ll P_T$, the approximate result will be:

$$P_{out2} = 1 - \exp\{-\frac{g_2\,(P_{AI} + P_R) + g_1 P_{BI}}{P_{BI} P_R} R_B \sigma^2\}. \quad (7)$$

Last when $\frac{P_{BI}}{P_{AI}+P_R} < z < \frac{P_{BI}+P_R}{P_{AI}}$, the outage probability depends on both two links, when the noise power $\sigma^2 \ll P_T$, the approximate result will be:

$$P_{out3} = 1 - \exp\{-\frac{g_2 P_{AI} + P_R}{P_{BI} P_R} R_B - \frac{g_1\,(P_{BI} + P_R)}{P_{AI} P_R} R_A\}. \quad (8)$$

To minimize the outage probability (6)(7)(8) will be the target of next subsection.

B. *Power Allocation*

In this subsection, we discuss the source nodes power allocation. In order to be more universal, we assume the energy harvesting efficiency of the relay is two fixed η_1 and η_2, which maybe the same in some system. R_A, R_B and σ^2 are fixed in certain communication system. Therefore our target can be expressed as follows.

The common conditions are (2) and

$$P_R = \eta_1 g_1 P_A + \eta_2 g_2 P_B. \quad (9)$$

These two expressions are the common condition of all the problems below.

For $\frac{P_{BI}}{P_{AI}+P_R} \geqslant z$, the problem will be (case I):

$$\min_{P_A, P_B} \frac{g_1\left((1-\eta_2)\,P_B + P_R\right) + g_2(1-\eta_1)P_A}{(1-\eta_1)P_A P_R}. \quad (10)$$

For $\frac{P_{AI}}{P_{BI}+P_R} \geqslant \frac{1}{z}$, the problem will be (case II):

$$\min_{P_A, P_B} \frac{g_2\left((1-\eta_1)\,P_A + P_R\right) + g_1(1-\eta_2)P_B}{(1-\eta_2)P_B P_R}. \quad (11)$$

For $\frac{P_{BI}}{P_{AI}+P_R} < z < \frac{P_{BI}+P_R}{P_{AI}}$, the problem will be (case III):

$$\begin{aligned}\min_{P_A,P_B}\{&\frac{g_2\left((1-\eta_1)P_A+P_R\right)}{(1-\eta_2)\,P_B P_R}R_B\\&+\frac{g_1\left((1-\eta_2)\,P_B+P_R\right)}{(1-\eta_1)P_A P_R}R_A\}.\end{aligned} \tag{12}$$

First, we discuss case III which looks like the most complex. For traditional three point relaying model, this case is the most possible which the power allocation point would be in, when difference of the channel situations for A and B is not big. Due to the outage probability in this situation considered both two links $A \to R \to B$ and $B \to R \to A$. However, in energy harvesting situation, we can greatly simplify this problem.

Proposition 1:For case III, approximately, the power allocation point will be:

$$\left(\frac{P_B}{P_A}\right)_{\text{III}} = \frac{(1-\eta_1)}{(1-\eta_2)}\eta. \tag{13}$$

Proof: In this case, the range of power allocation depends on $\frac{P_{BI}}{P_{AI}+P_R} < z < \frac{P_{BI}+P_R}{P_{AI}}$, so if the range is very small, we can consider it as one point. From the common condition (11), $\eta_1 < 1$ and $\eta_2 < 1$, we can get:

$$\begin{aligned}P_R &= \eta_1 g_1 P_A + \eta_2 g_2 P_B\\&< g_1 P_A + g_2 P_B.\end{aligned} \tag{14}$$

In wireless communication, the channel loss can be tens of dB or more, so $P_R \ll P_A, P_B$. Thus the range can be:

$$\frac{P_{BI}}{P_{AI}+o\,(P_{AI})} < z < \frac{P_{BI}+o\,(P_{BI})}{P_{AI}}. \tag{15}$$

Approximately, this case can be one point $\frac{P_{BI}}{P_{AI}} = \eta$, substitute $P_{AI} = (1-\eta_1)P_A$ and $P_{BI} = (1-\eta_2)\,P_B$ into it, we get (13). Thus *Proposition 1* has been proofed. ■

Then for case I, this is a problem of convex optimization, we use lagrangian multiplier method with KKT condition [11] to get the result.

Proposition 2: For case I, approximately, the power allocation point will be:

$$\left(\frac{P_B}{P_A}\right)_{\text{I}} = \frac{z(1-\eta_1+\eta_1 g_1)}{1-\eta_2-z\eta_2 g_2}. \tag{16}$$

Proof: In case I, we define lagrangian function:

$$\begin{aligned}&L(P_A,P_B,P_R,\eta_1,\eta_2,a,b,c)\\&=\frac{g_1\left((1-\eta_2)\,P_B+P_R\right)+g_2(1-\eta_1)P_A}{(1-\eta_1)P_A P_R}\\&+a(\eta_1 g_1 P_A+\eta_2 g_2 P_B-P_R)+b\,(P_A+P_B-P_T)\\&+c\left(z-\frac{(1-\eta_2)P_B}{(1-\eta_1)P_A+P_R}\right).\end{aligned} \tag{17}$$

The problem becomes:

$$\begin{cases}\dfrac{\partial L}{\partial P_A}=0,\dfrac{\partial L}{\partial P_B}=0,\dfrac{\partial L}{\partial P_R}=0,\\ \dfrac{\partial L}{\partial a}=0,\dfrac{\partial L}{\partial b}=0,\\ c=0or\dfrac{\partial L}{\partial c}=0\end{cases} \tag{18}$$

If $c = 0$, use other equations in (18), we will get:

$$\begin{aligned}&(g_1^2\eta_1(1-\eta_2)+g_2(\eta_2 g_2-\eta_1 g_1)(1-\eta_1))P_A^2\\&+2g_1^2\eta_1(1-\eta_2)P_A P_B+g_1 g_2\eta_2(1-\eta_2)P_B^2\\&=0.\end{aligned} \tag{19}$$

The left side of equal sign is always greater than zero, so abandon this condition.

If $\frac{\partial L}{\partial c} = 0$, use equation $\frac{\partial L}{\partial P_A} = 0$, $\frac{\partial L}{\partial P_B} = 0$, $\frac{\partial L}{\partial P_R} = 0$ and $\frac{\partial L}{\partial a} = 0$, $\frac{\partial L}{\partial b} = 0$, we can get (16). ■

For case II, it is symmetrical with the case I, use replacements as $P_A \leftrightarrow P_B$, $\eta_1 \leftrightarrow \eta_2$, $g_1 \leftrightarrow g_2$ and $z \leftrightarrow \frac{1}{z}$, we get the result:

$$\left(\frac{P_B}{P_A}\right)_{\text{II}} = \frac{z-z\eta_1-\eta_1 g_1}{1-\eta_2+\eta_2 g_2}. \tag{20}$$

Last, we give the conclusion of this subsection. The outage probability can be get in three cases, substituting (16) into (10), (20) into (11) and (13) into (12), choosing the minimum one as the best power allocation.

IV. Simulations

In this section, some numerical simulation results are given to confirm the theoretical results. Comparing these results we can find that the energy harvesting efficiencies have s great influence on the power allocation. In this section, we define $SNR = P_t/\sigma^2$, which means the total source nodes signal power to noise rate.

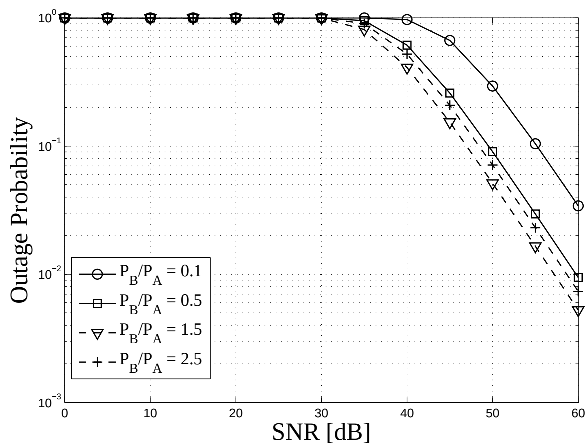

Fig. 2. Outage probability versus SNR when $\eta_1 = \eta_2 = 0.6$.

Because the energy harvesting relay needs a certain amount of power to broadcast signals, we set all the simulation in this section as short distance communication. Assuming a low pass loss as $g_2 = 10^{-7}$, $g_1 = 10^{-7}$, and setting the rate of each source node as $r_A = 1$ bits per second, $r_B = 1.5$ bits per second.

Fig. 2 shows the outage probability of different power allocations. In this simulation, the energy harvesting efficiencies are set as $\eta_1 = \eta_2 = 0.6$. From the result in subsection 3.2,

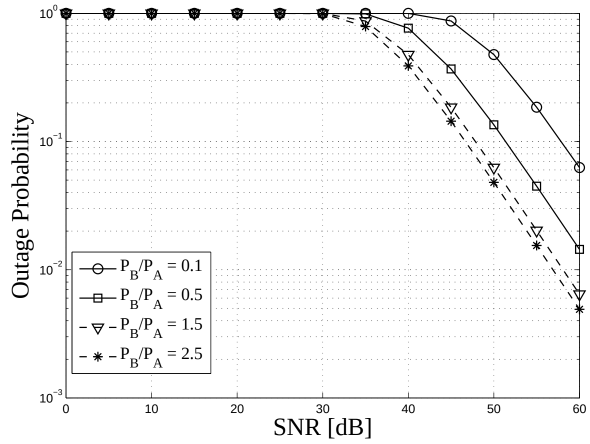

Fig. 3. Outage probability versus SNR when $\eta_1 = 0.5, \eta_2 = 0.6$.

$P_B/P_A = 1.5$ would be the best power allocation point, and the simulation result confirms it.

Fig. 3 shows the outage probability of different power allocations in another pair of η_1 and η_2. In this simulation, the energy harvesting efficiencies are set as $\eta_1 = 0.5$ and $\eta_2 = 0.7$, and other parameters are the same as Fig. 2. Thus, from the result in subsection 3.2, the power allocation point will be $P_B/P_A = 2.5$. This Fig confirms it.

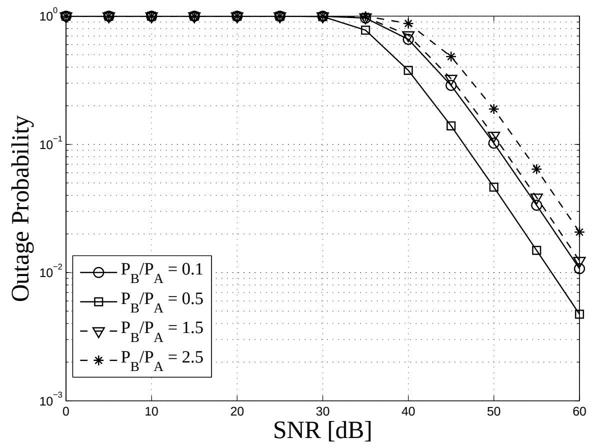

Fig. 4. Outage probability versus SNR when $\eta_1 = 0.8, \eta_2 = 0.4$.

Last, we set $\eta_1 = 0.8$ and $\eta_2 = 0.4$. Then as in Fig. 4, the best power allocation point is $P_B/P_A = 0.5$. From the three cases of different energy harvesting efficiencies above, we find that the best power allocation point drifts when η_1 and η_2 change. Thus there may be a best combination of power allocation and energy harvesting efficiencies that will gets the minimum outage probability. This will be talked in our next paper, considering the joint optimization of power allocation and energy harvesting efficiencies.

V. CONCLUSION

In this paper, closed form results for the outage probability of two-way relaying networks were introduced. The problem encountered when applying energy harvesting on two-way relaying networks. The correct form results for energy harvesting two-way relay is given. To minimize the outage probability, we derive closed form results for the source nodes power allocation with relay energy harvesting efficiency η_1 and η_2. Then the simulation results confirms our analytical results, and show that the energy harvesting efficiencies have s great influence on the power allocation.

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