Practicable Scheme to Generate the Three Photon Entangled State with Linear Optical Elements

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***Abstract*— This paper proposed practicable scheme for generation of the three photon entangled states without any non-linear optical process. The three-photon entanglement can be achieved by using linear optical elements with post selection based on the photons interferometry. The success probability of proposed method is 3/16, higher than the previous methods using the linear optical system.**

***Keywords*— Entangled photon, Photon interference, Optical element, Beam splitter**

I. Introduction

Entanglement is a non-local more characteristic with higher correlation than classical systems. The entangled states can be used for quantum information and communication such as quantum key distribution, teleportation and quantum computing. These applications require a large number of entangled state from primary resource. There are various methods to generate entangled state of two or more photon such as atomic, optical nonlinear crystal, optical fiber, semiconductors and optical linear elements. Most recently, the semiconductor and atomic are potential resources for the generation of entangled photon in the future. However, these technique are still very difficult to generate and maintain the entangled photon state such fragile environments. Parametric down conversion (PDC) in optical nonlinear crystals has been the most popular and powerful method of obtaining entangled photons. The pump light is converted into two lights (signal and idler) in a crystal with optical non-linearity. For example, the generation of polarization entangled photons using PDC with post selection procedure based on the photons interferometry as in [1]-[9]. Next, Kwiat et al. proposed the polarization entanglement was generated by a PDC crystal without the use of post selection as in [10]-[11]. Another method applying PDC to place nonlinear crystal in an interferometer such as Mach Zehnder and Sagnac in [12]-[22]. In addition, the optical linear elements is alternative technology for using to generate entangled photon which the capital cost of research lower another method. For instance, in [23] is proposed a scheme to produce an entangled four photon with linear optical elements which the entangled source is generated the initial state for system. The scheme for create four photon W states has been proposed in [24]. Next, in [25] was shown that the Greenberger–Horne–Zeilinger (GHZ) state of N photon was produced by simple linear optical elements. The optical gate for the generating to GHZ and W states with linear optical elements has been proposed by paper in [26]-[27]. However, there are some methods refers the physical of nonlinear crystals to an initially entangled source. For example, the entanglement source was used by paper in [23], [26] to create four photon entangled states.

This paper emphasizes to enhance generating for three entangled photon by using the linear optical elements with post selection based on the photons interferometry. Hence, the technical challenge of method is the generation of entangled photons without the PDC in nonlinear optical crystals. In addition, practicable scheme can be achieved in applications of quantum key distribution, teleportation and quantum computation.

II. Proposed Method

This paper proposed feasible scheme for generation of three photon entangled state as shown in Figure1. The method can be achieved by using linear optical elements with post selection based on the photons interferometry. The scheme is composed of single photon source, two beam splitters (50:50) and single photon detectors. The specific input state was prepared by single photon sources with one photon and two photons which both are injected into first beam splitter (BS1) from mode 1 and 2 respectively. There are four possible after BS1: (a) all photons are appears in mode 4, (b) one photon and two photons are appears in mode 3 and 4, (c) two photons and one photon are appears in mode 3 and 4, (d) all photons are appears in mode 3 respectively. One output of BS1 is divided into two modes of second beam splitter (BS2). Finally, the generating of three-photon entangled photon is successful when each of spatial mode has one photon with perfect post selection.

In consideration, the input states are defined by $|H\rangle_1$ and $|HV\rangle_2$ which the number in the subscript is the spatial mode of input state. The action of BS on horizontal polarization (H) can be described by the creation operators as

$$\hat{a}^{\dagger}_{1H}=\frac{1}{\sqrt{2}}(\hat{a}^{\dagger}_{3H}+\hat{a}^{\dagger}_{4H}) \ , \ \hat{a}^{\dagger}_{2H}=\frac{1}{\sqrt{2}}(\hat{a}^{\dagger}_{3H}-\hat{a}^{\dagger}_{4H})$$

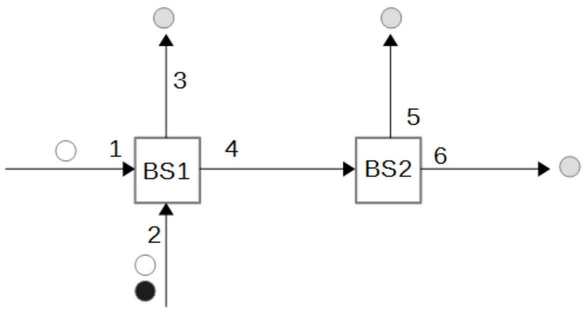

Figure 1. The schematic diagram of entangled three photon generation : one photon in the polarization state from mode 1 and two photons in the polarization state are injected into the BS1.

Similarly, the action of BS on vertical polarization (V) was denoted by
$\hat{a}^{\dagger}_{1V}=\frac{1}{\sqrt{2}}(\hat{a}^{\dagger}_{3V}+\hat{a}^{\dagger}_{4V})$, $\hat{a}^{\dagger}_{2V}=\frac{1}{\sqrt{2}}(\hat{a}^{\dagger}_{3V}-\hat{a}^{\dagger}_{4V})$.

Where $\hat{a}^{\dagger}_{jH}$ and $\hat{a}^{\dagger}_{jV}$ are the photon creation operators in H and V polarization for mode j. Therefore, the output states after BS1 is according to

$$\begin{aligned}|1_H\rangle_1|1_H1_V\rangle_2&=\frac{1}{\sqrt{2}}(\hat{a}^{\dagger}_{1H}\hat{a}^{\dagger}_{2H}\hat{a}^{\dagger}_{2V})|00\rangle_{3,4}\\&=\frac{1}{2\sqrt{2}}(\hat{a}^{\dagger}_{3H}+\hat{a}^{\dagger}_{4H})(\hat{a}^{\dagger}_{3H}-\hat{a}^{\dagger}_{4H})(\hat{a}^{\dagger}_{3V}-\hat{a}^{\dagger}_{4V})|00\rangle_{3,4}\\&=\frac{1}{2\sqrt{2}}|2_H1_V,0\rangle_{3,4}-\frac{1}{2\sqrt{2}}|2_H,1_V\rangle_{3,4}+\\&\quad\frac{1}{2\sqrt{2}}|0,2_H1_V\rangle_{3,4}-\frac{1}{2\sqrt{2}}|1_V,2_H\rangle_{3,4}+\\&\quad\frac{1}{2}|1_H1_V,1_H\rangle_{3,4}+\frac{1}{2}|1_H,1_H1_V\rangle_{3,4}\end{aligned}\tag{1}$$

The method of generation is successful when each of the output mode 3, 5 and 6 has one photon. Therefore, there are two terms which are useful for the generation of entangled three photon as

$\frac{1}{2\sqrt{2}}|1_V,2_H\rangle_{3,4}$ and $\frac{1}{2}|1_H,1_H1_V\rangle_{3,4}$.

The states $|2_H\rangle_4$ and $|1_H1_V\rangle_4$ are injected into the BS2 which the output state can be given by

$$|2_H\rangle_4=\frac{1}{2}|2_H,0\rangle_{5,6}+\frac{1}{\sqrt{2}}|1_H,1_H\rangle_{5,6}+\frac{1}{2}|0,2_H\rangle_{5,6}\tag{2}$$

$$\begin{aligned}|1_H1_V\rangle_4=&\frac{1}{2}|1_H1_V,0\rangle_{5,6}+\frac{1}{2}|1_H,1_V\rangle_{5,6}+\frac{1}{2}|1_V,1_H\rangle_{5,6}+\\&\frac{1}{2}|0,1_H1_V\rangle_{5,6}\end{aligned}\tag{3}$$

After the BS2, one photon of the output mode 5 and 6 are useful for generation of entangled three photon. Therefore, the output state becomes the entangled three photon can be achieved by post selection as follows

$$|1_H\rangle_1|1_H1_V\rangle_2=\frac{1}{4}(|VHH\rangle_{356}+|HVH\rangle_{356}+|HHV\rangle_{356})\tag{4}$$

Similarly, if one photon in the polarization state $|H\rangle_1$ and two photons in the polarization state $|VV\rangle_2$ are injected into BS1. The output state can be described by

$$\begin{aligned}|1_H\rangle_1|1_V1_V\rangle_2=&\frac{1}{2\sqrt{2}}|1_H2_V,0\rangle_{3,4}+\frac{1}{2}|1_H1_V,1_V\rangle_{3,4}+\\&\frac{1}{2\sqrt{2}}|1_H,2_V\rangle_{3,4}+\frac{1}{2\sqrt{2}}|0,1_H2_V\rangle_{3,4}+\\&\frac{1}{2\sqrt{2}}|2_V,1_H\rangle_{3,4}-\frac{1}{2}|1_V,1_H1_V\rangle_{3,4}\end{aligned}\tag{5}$$

The method of generation is successful when choose the events that there are one photon in mode 3, 5 and 6. Two terms of output state are useful for the generation of entangled three photon as $\frac{1}{2\sqrt{2}}|1_H,2_V\rangle_{3,4}$ and $\frac{1}{2}|1_V,1_H1_V\rangle_{3,4}$.

The states $|2_V\rangle_4$ and $|1_H1_V\rangle_4$ are injected into the BS2 which the output state can be given by

$$|2_V\rangle_4=\frac{1}{2}|2_V,0\rangle_{5,6}+\frac{1}{\sqrt{2}}|1_V,1_V\rangle_{5,6}+\frac{1}{2}|0,2_V\rangle_{5,6}\tag{6}$$

$$\begin{aligned}|1_H1_V\rangle_4=&\frac{1}{2}|1_H1_V,0\rangle_{5,6}+\frac{1}{2}|1_H,1_V\rangle_{5,6}+\frac{1}{2}|1_V,1_H\rangle_{5,6}+\\&\frac{1}{2}|0,1_H1_V\rangle_{5,6}\end{aligned}\tag{7}$$

Consequently, the output state becomes the entangled three photon can be achieved by post selection as follows

$$|1_H\rangle_1|1_V1_V\rangle_2=\frac{1}{4}(|HVV\rangle_{356}+|VVH\rangle_{356}+|VHV\rangle_{356})\tag{8}$$

In addition, one photon in the polarization state $|V\rangle_1$ and two photons in the polarization state $|HH\rangle_2$ are input for BS1. The output state can be described by

$$\begin{aligned}|1_V\rangle_1|1_H1_H\rangle_2=&\frac{1}{2\sqrt{2}}|1_V2_H,0\rangle_{3,4}-\frac{1}{2}|1_V1_H,1_H\rangle_{3,4}+\\&\frac{1}{2\sqrt{2}}|1_V,2_H\rangle_{3,4}-\frac{1}{2}|1_H,1_V2_H\rangle_{3,4}+\\&\frac{1}{2\sqrt{2}}|0,1_V2_H\rangle_{3,4}+\frac{1}{2\sqrt{2}}|2_H,1_V\rangle_{3,4}\end{aligned}\tag{9}$$

Two terms of output state are useful for the generation of entangled three photon as follows

$\frac{1}{2\sqrt{2}}|1_V,2_H\rangle_{3,4}$ and $\frac{1}{2}|1_H,1_V1_H\rangle_{3,4}$.

The states $|2_H\rangle_4$ and $|1_V1_H\rangle_4$ are injected into the BS2 which the output state can be written as

$$|2_H\rangle_4=\frac{1}{2}|2_H,0\rangle_{5,6}+\frac{1}{\sqrt{2}}|1_H,1_H\rangle_{5,6}+\frac{1}{2}|0,2_H\rangle_{5,6}\tag{10}$$

$$|1_V 1_H\rangle_4 = \frac{1}{2}|1_V 1_H, 0\rangle_{5,6} + \frac{1}{2}|1_V, 1_H\rangle_{5,6} + \frac{1}{2}|1_H, 1_V\rangle_{5,6} + \frac{1}{2}|0, 1_V 1_H\rangle_{5,6} \quad (11)$$

Consequently, the output state becomes the entangled three photon can be achieved by post selection as follows

$$|1_V\rangle_1 |1_H 1_H\rangle_2 = \frac{1}{4}(|VHH\rangle_{356} + |HHV\rangle_{356} + |HVH\rangle_{356}) \quad (12)$$

The success probability of generating three photon entangled is 3/16 with a perfect post selection. The results of method will evolve as follows Eq.(4), Eq.(8) and Eq.(12).

III. Conclusions

This paper presented the scheme for generation of three entangled-photon without any non-linear optical process. The three entangled-photon can be achieved by post selection based on photon interferometry at the beam splitter. The success probability of generating three entangled-photon is 3/16 and the method was compared with the previous method as shown in Table 1. This proposed method used only two beam splitters while another method used more elements for generation of three entangled-photon. However, the modified technique is depended on the theoretical of quantum mechanics for the generation of three entangled-photon which is the physical mechanism.

TABLE 1. COMPARISON OF THE SUCCESS PROBABILITY FOR PREPARING OF GENERATE TO THREE PHOTON ENTANGLED STATE.

Paper	The generation of three photon entangled state with linear optical elements	
	The photon entangled state	Success probability
In [26]	$\frac{1}{4}(|HHV\rangle_{456} + |HVH\rangle_{456} + |VHH\rangle_{456})$	3/16
In [23]	$\frac{1}{\sqrt{3}}(|HHV\rangle_{324} + |HVH\rangle_{324} + |VHH\rangle_{324})$	2/27
In [27]	$\frac{-\sqrt{2}}{8}(|VHH + HVH + HHV\rangle_{567} + |VHH + HVH + HHV\rangle_{568} + |VHH + HVH + HHV\rangle_{578} + |VHH + HVH + HHV\rangle_{678})$	3/8
Proposed method	$\frac{1}{4}(|VHH\rangle_{356} + |HVH\rangle_{356} + |HHV\rangle_{356})$, $\frac{1}{4}(|HVV\rangle_{356} + |VVH\rangle_{356} + |VHV\rangle_{356})$, $\frac{1}{4}(|VHH\rangle_{356} + |HHV\rangle_{356} + |HVH\rangle_{356})$	3/16

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