Analytical Description of Chromatic Dispersion Effect on Signal Propagation in the Time Domain

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***Abstract* —The investigation of chromatic dispersion effect on pulse propagation is of interest in high-speed optical transmission systems. But the chromatic dispersion effect hasn't an acceptable analytical description in the time domain. The analytical model of the dispersion effect in the time domain using a quadratic function approximation of nonlinear part of the propagation constant and the Fresnel integrals is proposed in this paper. It is shown that the obtained model is universal and it has a tunable accuracy. A simple method of estimating the memory of an optical channel is proposed. The analytical model of signal propagation in an optical channel by means of sequential generation of pairs of echo-signals is described in the article.**

***Keyword*—Analytical model, approximation, dispersion, Frensel integrals, propagation constant, time domain, echo-signal, memory of channel**

I. INTRODUCTION

AT the present time, high-speed optical transmission systems are in an active development and the investigation of the chromatic dispersion effect is one of most interest of issues. The propagation of pulses through an optical fiber, which is a dispersive medium, is well explored in [1]–[7].

$$\frac{\partial A}{\partial z}+\beta_1\frac{\partial A}{\partial t}+\frac{j\beta_2}{2}\frac{\partial^2 A}{\partial t^2}-\frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3}=0 \tag{1}$$

where $A(z,t)$ is the slowly varying pulse envelope and β_m are a parameters described in [1] .

The differential equation (1) should be solved if a pulse form in the time domain is required to find. And, this equation should be solved individually for of all kinds of pulses. Besides, a direct solution may be difficult, whereas an analytical equation is impossible to get using the fast Fourier transform. But the analytical equation may be of use in some cases, e.g. signal processing in the time domain.

In this paper, a simple and universal analytical dispersion model in the time domain will be found.

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II. DISPERSION THEORY

In the Dispersion Theory [1], pulse propagation can be written in the form

$$\frac{\partial\tilde{A}}{\partial z}=-j\beta(f)\tilde{A} \tag{2}$$

where $\tilde{A}(z,f)$ is a signal spectrum at a distance z, and $\beta(f)$ is the propagation constant which has a frequency dependence

$$\beta(f)=\frac{2\pi f}{c}\cdot n(f) \tag{3}$$

Here, c is the velocity of light in vacuum and n is a refractive index which defined by the Sellmeier equation [8].

All nonlinear effects and the attenuation have been excluded in equation (2).

The solution of the equation is

$$\tilde{A}(z,f)=\tilde{A}(0,f)e^{-j\beta(f)z} \tag{4}$$

It completely described the effect of the chromatic dispersion on a signal.

Therefore, the transfer function of the dispersive medium is

$$H(f)=e^{-j\beta(f)z} \tag{5}$$

III. DISPERSION IN THE TIME DOMAIN

As seen from Figure 1, the propagation constant has a substantial linear part. Therefore, we can represent it as

$$\beta(f)=\beta_{ln}(f)+\beta_{nl}(f) \quad (6)$$

where β_{ln} is a linear part and β_{nl} is a nonlinear part of the propagation constant.

Since the entire optical spectrum is not of interest an operating band of signal will be considered further, i.e. $f \in [f_l; f_h]$ where, f_h is the higher frequency, and f_l is the lower frequency.

A. Linear Part of the Propagation Constant

The influence of β_{ln} causes signal delay in the time domain. β_{ln} can be represented in form

$$\beta_{ln}(f)=kf+p \quad (7)$$

And parameters *k* and *p* are

$$\begin{cases} k=\dfrac{\beta(f_h)-\beta(f_l)}{f_h-f_l} \\ p=\dfrac{\beta(f_l)f_h-\beta(f_h)f_l}{f_h-f_l} \end{cases} \quad (8)$$

If the nonlinear part is omitted, a pulse envelope in the time domain will be obtained in the form

$$A(z,t)=A\left(0,t-\frac{kz}{2\pi}\right)e^{-jpz} \quad (9)$$

B. Nonlinear Part of the Propagation Constant

The nonlinear part can be obtained as

$$\beta_{nl}(f)=\beta(f)-\beta_{ln}(f) \quad (10)$$

As seen from Fig. 2, it is look like quadratic function. Therefore, it can be represented as

$$\beta'_{nl}(f)=af^2+bf+c \quad (11)$$

where *a*, *b* and *c* are

$$\begin{cases} a=-4\left[\beta(f_c)-\beta_{ln}(f_c)\right]\dfrac{1}{(f_h-f_l)^2} \\ b=4\left[\beta(f_c)-\beta_{ln}(f_c)\right]\dfrac{(f_h+f_l)}{(f_h-f_l)^2} \\ c=-4\left[\beta(f_c)-\beta_{ln}(f_c)\right]\dfrac{f_h f_l}{(f_h-f_l)^2} \end{cases} \quad (12)$$

Here, f_c is a central frequency of the band.

Actually, β_{nl} is not a quadratic function. This function has some cubic part. Accordingly, there is a deviation, as shown in Fig. 3. But β_{nl} can not be approximated with a cubic function because it will obstruct further calculations.

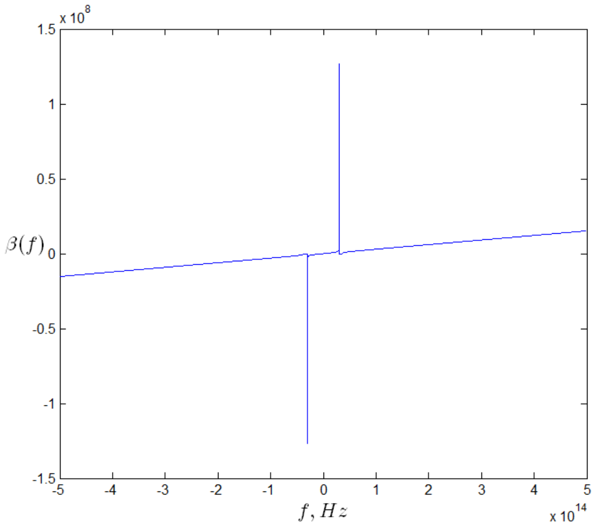

Fig. 1. Frequency dependence of the propagation constant.

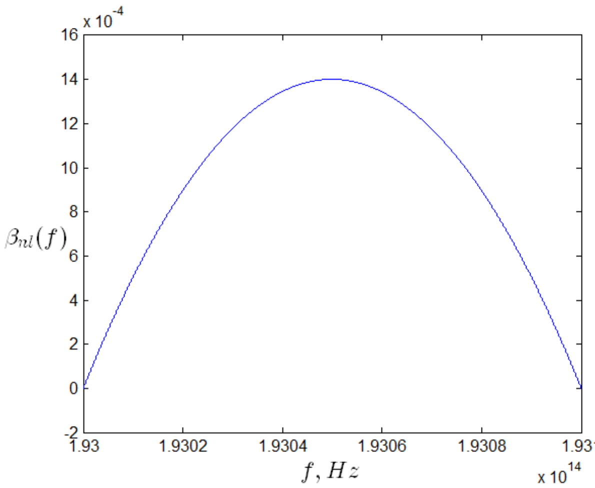

Fig. 2. A nonlinear part of the propagation constant in the 100 GHz operating band.

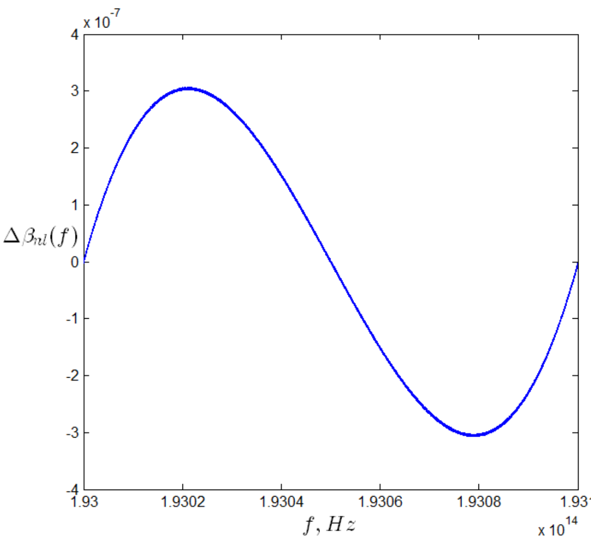

Fig. 3. An absolute error of the quadratic function approximation of nonlinear part of the propagation constant.

To minimize the deviation, the entire operating band can be divided into two. And then, the deviation function can be approximated with a quadratic function in each of sub-bands

$$\Delta\beta_{nl}(f)=\begin{cases} m_1 f^2+n_1 f+k_1,\ f\in[f_l;f_c] \\ m_2 f^2+n_2 f+k_2,\ f\in[f_c;f_h]\end{cases} \tag{13}$$

and

$$\Delta\beta_{nl}(f)=\beta_{nl}(f)-\beta'_{nl}(f) \tag{14}$$

Parameters m_1, n_1, k_1, m_2, and k_2 can be found with (12), where f_h, f_l and f_c is a higher, lower and the central frequency of the each of the ranges, respectively.

A new approximation is

$$\beta''_{nl}(f)=\begin{cases}(a+m_1)f^2+(b+n_1)f+(c+k_1),\ f\in[f_l;f_c] \\ (a+m_2)f^2+(b+n_2)f+(c+k_2),\ f\in[f_c;f_h]\end{cases} \tag{15}$$

Deviations in each of the ranges will be similar to the deviation of the first approximation. Therefore, a deviation in the entire band can be represented by four quadratic functions in four intervals.

It was found that improving of the approximation can be continued in a similar way until the error value becomes sufficient.

C. Representation of Chromatic Dispersion in the Time Domain

The transfer function of the dispersive medium can be represented as a product of two parts

$$H(f)=e^{-j\beta_{ln}(f)z}\cdot e^{-j\beta_{nl}(f)z} \tag{16}$$

First part is a linear part which described in the time domain by (9). Second part is a nonlinear part which causes a pulse distortion. Let us expand it into a Fourier series to describe it in the time domain

$$H(f)\approx e^{-j\beta_{ln}(f)z}\cdot\sum_{n=-\infty}^{\infty}c_n e^{j\frac{\pi fn}{L}} \tag{17}$$

where

$$c_n=\frac{1}{2L}\int_{f_l}^{f_h}e^{-j\beta_{nl}(f)z}e^{-j\frac{\pi fn}{L}}df \tag{18}$$

and L is a half of the entire band.

Because $\beta_{nl}(f)$ is a quadratic function c_n can be expressed in terms of the Fresnel integrals [9]

$$\begin{aligned} c_n=&\frac{e^{-j\left(\frac{(bzL+\pi n)^2}{-4azL^2}+cz\right)}}{2L\sqrt{-az}} \\ &\cdot\left[C\left(\sqrt{-az}f_h-\frac{bzL+\pi n}{2\sqrt{-az}L}\right)-C\left(\sqrt{-az}f_l-\frac{bzL+\pi n}{2\sqrt{-az}L}\right)\right. \\ &\left.+jS\left(\sqrt{-az}f_h-\frac{bzL+\pi n}{2\sqrt{-az}L}\right)-jS\left(\sqrt{-az}f_l-\frac{bzL+\pi n}{2\sqrt{-az}L}\right)\right]\end{aligned} \tag{19}$$

Here, C(x) and S(x) are the Fresnel integrals.

For second approximation of nonlinear part of the propagation constant from (15) and for next approximations, (19) should be calculated in each of sub-bands (with constant L), and the results should be summarized then.

A signal spectrum at a distance z is

$$\tilde{A}(z,j\omega)=\tilde{A}(0,j\omega)H(j\omega) \tag{20}$$

or, with (17) and (7) is

$$\tilde{A}(z,j\omega)=\sum_{n=-N}^{N}c_n e^{-jpz}\tilde{A}(0,j\omega)e^{-j\omega\left(\frac{kz}{2\pi}-\frac{n}{2L}\right)} \tag{21}$$

To describe a signal in the time domain the inverse Fourier transformation [11] should be used.

Finally, in the time domain

$$A(z,t)=\sum_{n=-N}^{N}c_n e^{-jpz}A\left(0,t+\frac{n}{2L}-\frac{kz}{2\pi}\right) \tag{22}$$

where A is a pulse envelope at a distance z and at a time moment t. L is a half of a signal bandwidth. c_n are given by (19), and p and k are given by (8).

Equation (22) describes any signal at a distance z which is distorted by the chromatic dispersion.

IV. Propagation Process in Terms of the Echo-Signals Model

Model of Chromatic Dispersion Effect (22) allows representing a signal propagation process in terms of echo-signals.

The echo-signals model [11] describes a signal at a distance z as the sum of initial signal and N pairs of lagged and anticipatory echo-signals.

$$\begin{aligned} A(z,t)=&a_0A(0,t)+\sum_{n=1}^{N}a_nA(0,t+n\tau) \\ &+\sum_{n=1}^{N}a_{-n}A(0,t-n\tau)\end{aligned} \tag{23}$$

Here, $A(0,t)$ is initial signal, $A(0,t+n\tau)$ are anticipatory echo-signals, $A(0,t-n\tau)$ are lagged echo-signals and are some coefficients.

Expression (22) can be represented in the same form

$$A(z,t)=c_0e^{-jpz}A\left(0,t-\frac{kz}{2\pi}\right)+\sum_{n=1}^{N}c_ne^{-jpz}A\left(0,t+\frac{n}{2L}-\frac{kz}{2\pi}\right)+\sum_{n=1}^{N}c_{-n}e^{-jpz}A\left(0,t-\frac{n}{2L}-\frac{kz}{2\pi}\right) \quad (24)$$

Here, c_ne^{-jpz} corresponds to a_n, $\frac{1}{2L}$ corresponds to τ and $t-\frac{kz}{2\pi}$ corresponds to t in (23).

A. Memory of Optical Channel

The memory of channel allows estimating the number of pulses, which influence one another.

Interval between centers of two echo-signals is $\tau=\frac{1}{2L}$. Also, τ is interval between centers of two pulses. Therefore, the number of echo-signals equals the number of pulses, which influence one another. The number of echo-signals is defined by coefficients c_n, which can be called the impulse response of an optical channel if nonlinear effects are negligible. Therefore, the memory of an optical channel can be found by estimating values of c_n.

Example of the impulse response of an optical channel is shown in Fig. 4. The coefficients until certain n do not have a decreasing character and then begin decreasing rapidly.

Plotting of similar graphs allows simple finding the memory of an optical channel.

B. Propagation Process

Let us consider signal at distance *dz* at which there are two echo-signals only

$$A(dz,t)=a_0A(0,t)+a_1A(0,t+\tau)+a_{-1}A(0,t-\tau) \quad (25)$$

The entire line with length *z* can be divided to *N* sections with lengths *dz* as shown in Fig. 5.

Each section transforms the signal the same way as in (25): the input is $A([n-1]dz,t)$, the output is

$$A(ndz,t)=a_0A([n-1]dz,t)+a_1A([n-1]dz,t+\tau)+a_{-1}A([n-1]dz,t-\tau) \quad (26)$$

One section can be represented as FIR filter as shown in Fig. 6. Each section generates a new pair of echo-signals:

$$A(2dz,t)=\left[a_0^2+2a_1a_{-1}\right]A(0,t)+2a_0a_1A(0,t+\tau)+2a_{-1}a_0A(0,t-\tau)+a_1^2A(0,t+2\tau)+a_{-1}^2A(0,t-2\tau) \quad (27)$$

Considering that $a_1=a_{-1}$, signal at the distance *z* can be described by expression

$$A(Ndz,t)=\left[\sum_{k=0}^{K}c_{0k}a_0^{N-2k}a_1^{2k}\right]A(0,t)+\sum_{n=1}^{N}\left[\sum_{k=0}^{K}c_{nk}a_0^{N-(2k+n)}a_1^{2k+n}\right]A(0,t+n\tau)+\sum_{n=1}^{N}\left[\sum_{k=0}^{K}c_{nk}a_0^{N-(2k+n)}a_1^{2k+n}\right]A(0,t-n\tau) \quad (28)$$

Here, *N* is number of the sections, $K=\left\lfloor\frac{N-n}{2}\right\rfloor$ is the number of summand at *n*th echo-signal and c_{nk} are coefficients at the summands which calculated as follows

$$c_{nk}=\frac{N!}{k!(k+n)!(N-2k-n)!} \quad (29)$$

Besides, $\left[\sum_{k=0}^{K}c_{0k}a_0^{N-2k}a_1^{2k}\right]$ corresponds c_0 and $\left[\sum_{k=0}^{K}c_{0k}a_0^{N-2k}a_1^{2k}\right]$ corresponds $c_1=c_{-1}$ from (24).

As can be seen from the above, the propagation process in an optical channel can be represented as sequential generation of pairs of echo-signals after each section with length *dz*. It could be useful for investigation of nonlinear propagation in an optical channel because the model (28) is not requires performing the Fourier transform in distinction from the split-step Fourier method [1]. All of operations are performed in time domain.

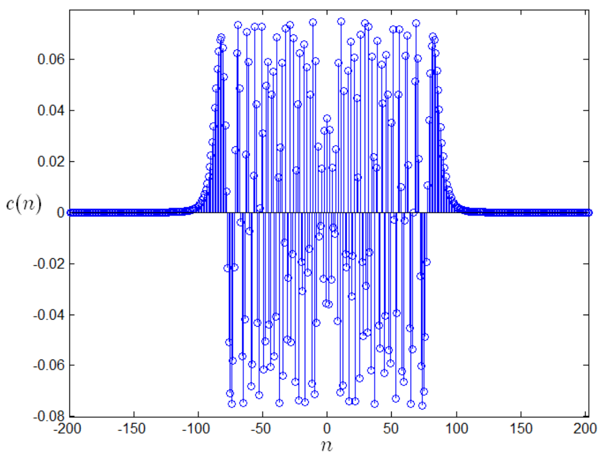

Fig. 4. The impulse response of an optical channel with 100 GHz operating band at the distance 100 km.

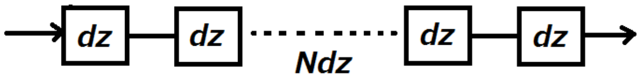

Fig. 5. The line with length *z* which is divided to *N* sections with lengths *dz*.

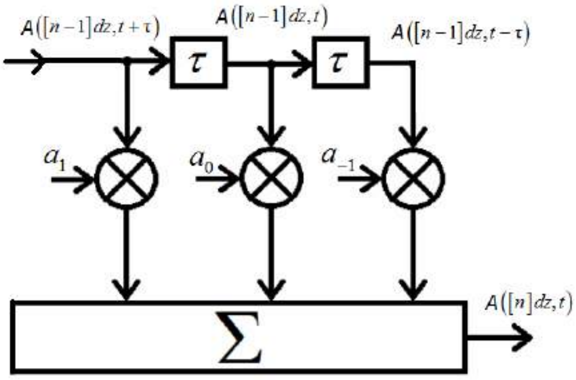

Fig. 6. Section *dz* as FIR filter.

V. Experimental Results

An accuracy of the obtained analytical model is defined by the number of the Fourier series coefficients N and by the number of the approximations as in (15). The first parameter is substantial in any conditions. The second is substantial only at considerable distances ≥ 1000 km and at signal bandwidths ≥ 100 GHz. Dependencies of the parameter N are

$$N(z) \approx z(km) \tag{30}$$

$$N(\Delta F) \approx \frac{\left[\Delta F(GHz)\right]^2}{100} \tag{31}$$

Some types of pulses calculated with the model are shown in Fig. 7, 8 and 9.

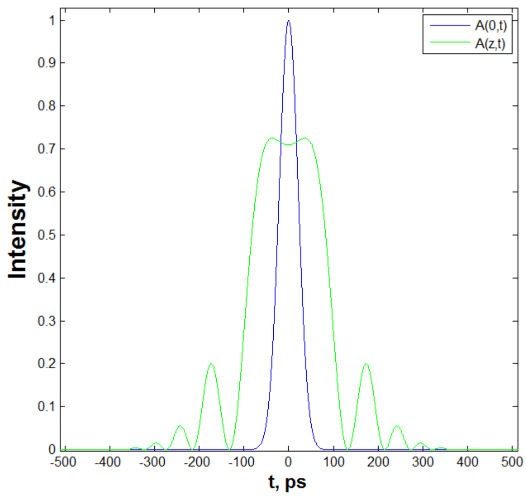

Fig. 7. Gaussian pulse with width 30 ps at the distance 155 km. The constant delay, which equaled 0.76 ms, has been compensated.

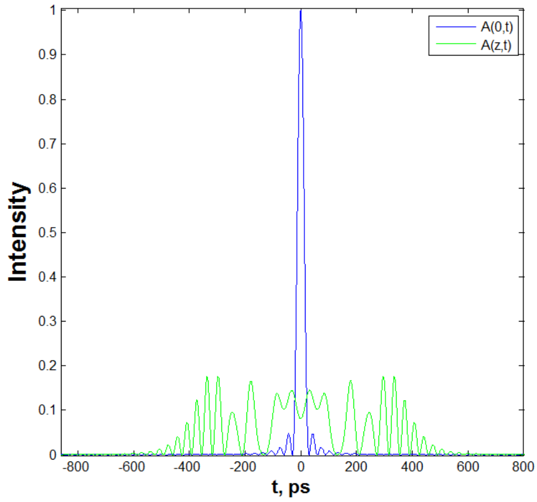

Fig. 8. Sinc pulse with width 30 ps at the distance 155 km. The constant delay, which equaled 0.76 ms, has been compensated.

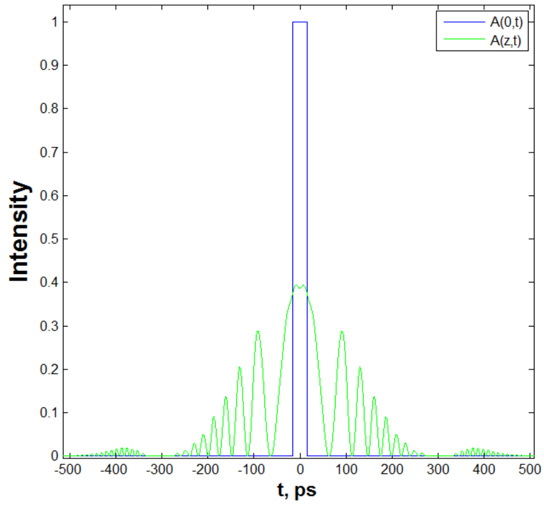

Fig. 9. Square pulse with width 30 ps at the distance 50 km. The constant delay, which equaled 0.24 ms, has been compensated.

VI. Conclusion

In this paper, an analytical model of dispersion effect in the time domain has been described. This model allows describe the chromatic dispersion effect on any signal, which propagates through optical fiber. The model has a tunable accuracy, so it is applicable in different areas such as signal propagation modeling, algorithms of dispersion compensation, etc.

It has been shown that the propagation process in an optical channel can be represented as sequential generation of pairs of echo-signals. The model is an analytical and described in time domain. The applications of this model are an optical signal propagation modeling which includes the nonlinear propagation and algorithms of dispersion compensation.

Furthermore, the method of estimating the memory of an optical channel is proposed in this article. It requires only the estimation of values of coefficients at each echo-signal.

A problem of the proposed models is a high computational complexity in the case of wide signal bandwidths and considerable distances. However, the main task of this model is the using it in an analytical equations. In this case, the computational complexity is not substantial.

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