A Cooperative Spectrum Sensing Algorithm Using Leading Eigenvector Matching

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***Abstract*—Cognitive radio emerged as a new trend to mitigate the severe spectrum scarcity problem. As an essential problem in cognitive radio, spectrum sensing has been discussed widely recently. Blind detection techniques that sense the presence of a primary user's signal without prior knowledge of the signal characteristics, channel and noise power attract more attention than non-blind detection. The sensing algorithms based on random matrix theory which are shown to outperform energy detection especially in case of noise uncertainty. In this paper, a sensing algorithm using leading eigenvector matching (LEM) is introduced into cooperative spectrum sensing process. LEM detector uses the feature blindly learned from feature learning algorithm (FLA) as prior knowledge. The LEM algorithm involves the correlation coefficient between feature learned and leading eigenvector of sample covariance matrix as the test statistic. In this paper, we also derive the closed-form expression of the threshold in order to achieve constant false alarm rate detection. Numerical simulations show that the proposed detection algorithm performs better than the MME detector and it does not suffer from a noise power uncertainty problem while also proving to be more robust against the correlation decrease between sensing nodes.**

***Keyword*—Cognitive Radio, spectrum sensing, sample covariance matrix, leading eigenvector matching, feature learning.**

I. INTRODUCTION

COGNITIVE radio (CR) differs from conventional radio systems and is considered as an effective method to mitigate the spectrum scarcity problem. In CR, cognitive user (CU) is aware of the electromagnetic environment around it and accesses the spectrum underutilized by primary user's (PU) accordingly [1]. Spectrum sensing is an essential problem in CR which can detect the PU's signal presence and it has been widely discussed in recent decade [2]. It is simple to detect signal when the signal to noise (SNR) is high, but in practice sensing the presence of PU's signal becomes demanding because of the low SNR and shadow fading. Spectrum sensing algorithms existed can be divided into non-blind detector and blind detector according to whether it requires prior knowledge about the signal and the channel characteristics. Non-blind techniques (such as matched filter detection and cyclostationary feature detection [3]) that rely on prior knowledge give a better performance, but it is difficult to acquire prior knowledge in practice. On the other hand, blind sensing (e.g. energy detection) that do not require prior knowledge is flexible in their application.

Aforementioned detection algorithms are single-node sensing methods whose performances fall down quickly because of the multipath fading and hidden terminal problems, so cooperative spectrum sensing algorithms attract more attention. The cooperative spectrum sensing algorithms based on random matrix theory (RMT) were shown to outperform classical methods as a blind detector, especially in case of noise uncertainty which is the main disadvantage of energy detection. Most of the algorithms based on RMT utilize the differences between the distributions of eigenvalues of sample covariance matrix under H_0 and H_1 , including maximum and minimum eigenvalue (MME) [4], energy with minimum eigenvalue (EME) [5], maximum eigenvalue detection (MED) [6]. However, the algorithms based on RMT suffer from the correlation problem, i.e., its perceived performance decreases quickly when the correlation between sensing nodes decreases.

Besides eigenvalues, eigenvector is another characteristic of the covariance matrix. Feature template matching (FTM) [7] has been proposed as a single-node spectrum sensing technique which is based on the leading eigenvector and shown that it performs better than MME and the covariance absolute value (CAV) algorithms. Multiple feature matching (MFM) [8] algorithm applied the FTM algorithm in MIMO system. But the methods above did not derive the closed-form expression of the threshold and were limited in single-node spectrum sensing.

In this paper, a cooperative spectrum sensing algorithm using leading eigenvector matching (LEM) is introduced. LEM detector uses the feature blindly learned from feature learning algorithm (FLA) as prior knowledge. The correlation coefficient between feature learned and the leading eigenvector of sample covariance matrix serves as the test statistic for signal detection. The closed-form expression of the threshold is also derived in this paper. Simulation results show that the algorithm proposed is reasonable and LEM detector outperforms MME detector. It also do not suffer from a noise power uncertainty problem. Compared with MME detector, LEM detector is more robust against the decrease of correlation among the sensing nodes.

The rest of the paper is organized as follows: Sec. II

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reviews the sensing model and basic theory of the proposed algorithm. Sec. III deals with the description of LEM detector and the threshold derivation problem. Simulation results are presented and discussed in Sec. IV. Sec. V contains the conclusions.

II. System Model and Basic Theory

A. System Model

In the CR network, there is one PU and K CUs. Denote with $x_i(n)$ the n^{th} sample received by the i^{th} PU. There are two hypotheses and H_0 indicates that the PU's signal does not exist and H_1 denotes the signal exists. The received signal samples under two hypotheses show as follows:

$$x_i(n)=\begin{cases} w_i(n) & H_0 \quad i=1,2,...,K \\ s_i(n)+w_i(n) & H_1 \quad i=1,2,...,K \end{cases} \tag{1}$$

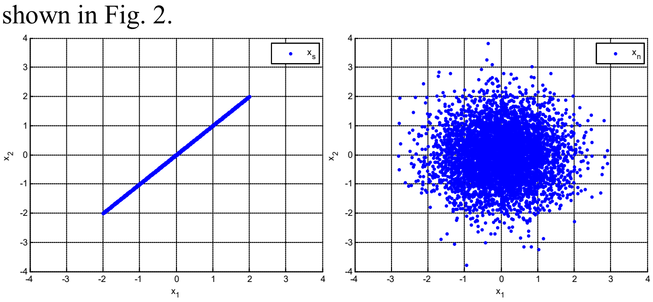
where $s_i(n)$ is the PU's signal received and $w_i(n)$ is the white Gaussian noise (WGN) with zero mean and variance σ^2. Let $X_i(n)=[x_i(n)\ x_i(n+1)\ ...\ x_i(n+N\text{-}1)]$ be a $1\times N$ vector containing N consecutive samples collected by the i^{th} CU. The j^{th} sensing segment constructed by the samples received is written as:

$$X_j=\begin{bmatrix} X_1(j) \\ X_2(j) \\ \vdots \\ X_K(j) \end{bmatrix}=\begin{bmatrix} x_1(j) & x_1(j+1) & \dots & x_1(j+N-1) \\ x_2(j) & x_2(j+1) & \dots & x_2(j+N-1) \\ \vdots & \vdots & \ddots & \vdots \\ x_K(j) & x_K(j+1) & \dots & x_K(j+N-1) \end{bmatrix} \tag{2}$$

The sample covariance matrix is $R_j=\frac{1}{N}X_jX_j^T$ and its leading eigenvector is φ_j.

B. Basic Theory

x_s indicates a 2×1 amplitude modulation (AM) signal vector and x_n indicates a 2×1 WGN vector. Denote with x_{s+n} a 2×1 AM signal with WGN vector, which means $x_{s+n}=x_s+x_n$. The elements of the vector are x_1 and x_2 respectively. Fig. 1 shows that the distribution of WGN is random, while x_s has the same characteristic angle with x_{s+n}, which means that their leading eigenvector is similar. The algorithm proposed uses the characteristic to distinguish signal from noise. The similar situation in three-dimension is shown in Fig. 2.



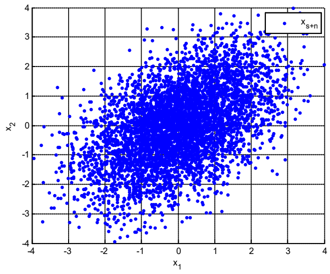


Fig. 1. The distribution under two-dimension

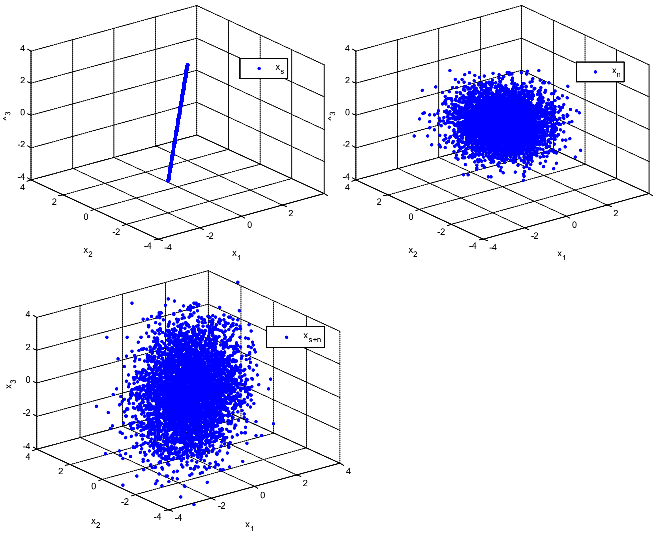


Fig. 2. The distribution under three-dimension

Mathematical theorem about the sensitivity of eigenvectors also explain the characteristic above, that is, the sensitivity of the eigenvector depends on the separation between the corresponding eigenvalue and other eigenvalues.

Under H_0, the sample covariance matrix approximates the diagonal matrix $\sigma^2 E$. The leading eigenvector is sensitive and random because the maximum eigenvalue equals to other eigenvalues, which means the similarity between two leading eigenvector of two sample covariance matrices is low. Instead, the leading eigenvector remains stable under H_1 because the maximum eigenvalue is much larger than other eigenvalues and the similarity is high.

According to the definition of the eigenvalues and eigenvectors, the relation among sample covariance matrix R, its maximum eigenvalue λ and leading eigenvector I can be written as the equation:

$$RI=\lambda I \tag{3}$$

While noise exists, it can be expressed as:

$$(R+\sigma^2 E)I=(\lambda+\sigma^2)I \tag{4}$$

It is shown that the leading eigenvector I of matrix R is also the leading eigenvector of $R+\sigma^2 E$. Therefore, I remains stable regardless of the change of noise variance, which is more robust against noise.

III. Detection Algorithm

A. Leading Eigenvector Matching Algorithm

Leading eigenvector is also called signal feature in pattern recognition and it has the greatest mutual information with original signal. Compared with the randomness of leading eigenvector of WGN, the leading eigenvector of WGN is more stable. If PU's signal exists, highly correlated leading eigenvector can be detected in consecutive sensing segments X_j and X_{j+1}. Due to the robustness of signal feature, it can be learned by blind FLA.

The feature φ_s can be learned blindly from J sensing segments by following steps:

1) Extract feature φ_j and φ_{j+1} from X_j and X_{j+1};
2) Compute correlation coefficient via cosine similarity formula:

$$\rho_{j,j+1} = \left| \frac{\left\langle \varphi_j, \varphi_{j+1} \right\rangle}{|\varphi_j||\varphi_{j+1}|} \right| = |\varphi_j^T \varphi_{j+1}| \quad (5)$$

3) J-1 correlation coefficients can be calculated from J sensing segments. If $\rho_{m,m+1} = \max_{j=1,2,\dots,J-1} \{\rho_{j,j+1}\}$, signal feature φ_s is learned as φ_{m+1}.

With the prior knowledge φ_s, we have LEM detector:

1) Compute the received signal sample covariance matrix $X_{current}$ and corresponding leading eigenvector $\varphi_{current}$;
2) Compute correlation coefficient $\rho_{s,current}$ between φ_s and $\varphi_{current}$;
3) H_1 is true if $\rho_{s,current} > \varepsilon$, where ε is the threshold determined by desired P_f.

Compared with MME detector, both of the algorithms need to solve eigenvector and eigenvalue problem and their time complexity is almost same. But the LEM detector need to learn the feature φ_s by FLA which requires extra computation and time. In practical applications, the feature can be learned ahead and stored in local fusion center memory.

B. Threshold

It is necessary to obtain the expression of the false-alarm probability. On the one hand, the false-alarm probability can be used to illustrate the performance of the detection algorithm. On the other hand, the threshold can be obtained by given target false-alarm probability. In the proposed algorithm, the probability of false alarm is defined as:

$$P_f = p(|\varphi_s^T \varphi_{current}| > \varepsilon | H_0) \quad (6)$$

Under H_0, we have the following result:

$$R_s, R_{current} \sim \frac{1}{S} wishart(S, \sigma^2 I) \quad (7)$$

where $wishart(\bullet)$ is Wishart distribution. Let φ_s and $\varphi_{current}$ be the leading eigenvectors of R_s and $R_{current}$, and the normalized covariance matrices are defined as:

$$\begin{aligned} C_s &= \frac{1}{\sigma^2} R_s, C_{current} = \frac{1}{\sigma^2} R_{current} \\ C_s, C_{current} &\sim \frac{1}{S} wishart(S, I) \end{aligned} \quad (8)$$

Let A and B be the matrices containing the normalized eigenvectors of C_s and $C_{current}$ respectively. We have $A = [a_1, a_2 \cdots a_K]$, $B = [b_1, b_2 \cdots b_K]$ where the eigenvectors are arranged in descending order. And the matrices A^T and B^T converge in distribution to Haar [9]. It is known that A, B and B^T are unitary matrices and we have following result:

$$f(A^T B) = f(A^T (B)^T) = f(A^T) \quad (9)$$

where $f(\bullet)$ is the Probability Density Function. Because $A^T B$ and A^T converge in the same distribution, the elements of the matrices also converge in the same distribution and we have following result:

$$f(|a_1^T b_1|) == f(|a_{11}|) \quad (10)$$

where a_{11} is the element at the first row and the first column in matrix A.

According to the property of unitary matrix, it can be known that a_{11}^2 converges in distribution to Beta with parameters $\alpha = \frac{1}{2}$ and $\beta = \frac{K-1}{2}$. The probability density function of $T = |a_1^T b_1| = |\varphi_s^T \varphi_{current}|$ can be written as:

$$f(T) = \frac{\Gamma(\frac{K}{2})}{\Gamma(\frac{1}{2})\Gamma(\frac{K-1}{2})} \frac{(1-T^2)^{(K-3)/2}}{T} \quad (11)$$

And we have the expression of false-alarm probability as follows:

$$\begin{aligned} P_f &= p(|\varphi_s^T \varphi_{current}| > \varepsilon | H_0) \\ &= \frac{\Gamma(\frac{K}{2})}{\Gamma(\frac{1}{2})\Gamma(\frac{K-1}{2})} \int_{\varepsilon}^{1} \frac{(1-T^2)^{(K-3)/2}}{T} dT \end{aligned} \quad (12)$$

Hence, we derive the decision threshold as a function of the false-alarm probability:

$$\varepsilon = \sqrt{F_{Beta}^{-1}(1-P_f, \frac{1}{2}, \frac{K-1}{2})} \quad (13)$$

where $F_{Beta}^{-1}(\cdot)$ is the inverse cumulative distribution function of Beta distribution. It is shown that the threshold is a function of the target P_f and the dimension of the covariance matrix K. So the algorithm proposed in this paper is a blind sensing algorithm that do not require any prior knowledge about the signal and the channel characteristics.

IV. SIMULATION

In this section, we present the simulation results to evaluate the performance of the proposed algorithm. The PU's signal is the AM signal whose carrier frequency is 702 KHz and the sample ratio is 4MHz. The numbers of the CUs and samples are 32 and 1000 respectively and the SNR is -20dB. The simulation results are obtained by 10,000 Monte Carlo trials.

A. Distribution of the test statistic

Fig. 3 shows the frequency distribution of the test statistic under two hypotheses, i.e., H_0 and H_1. It is shown that most of the test statistic under H_0 is less than 0.4 while the majority of the test statistic under H_1 is greater than 0.4. It is obvious that the test statistic under H_0 and H_1 can be separated well by a given threshold, e.g., 0.45, in this situation.

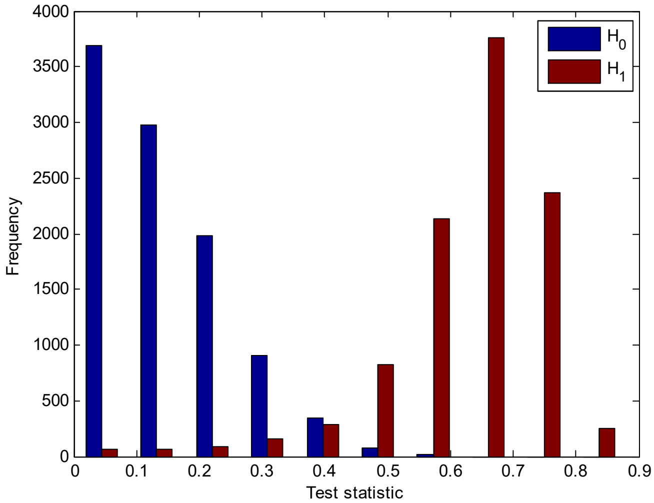


Fig. 3. Frequency distribution of the test statistic under H_0 and H_1

Fig. 4 presents the estimated and empirical cumulative distribution function (CDF) of the test statistic under H_0 respectively with different K. The accuracy of the estimated CDF determines the accuracy of the threshold to achieve target false-alarm probability. It is shown that the estimated CDF matches well with the empirical CDF with different K.

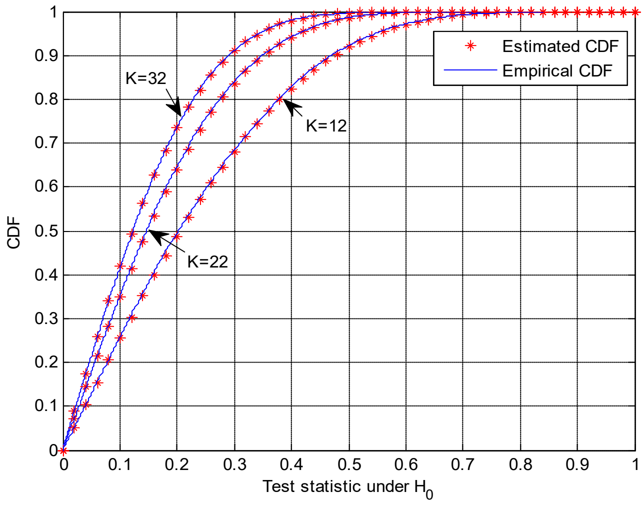


Fig. 4. Estimated and empirical CDF of the test statistic under H_0

B. Comparison of ROC for MME and LEM

MME algorithm is a classical cooperative spectrum sensing method based on the random matrix theory. Denote $\lambda_{\max}$ and $\lambda_{\min}$ with maximum and minimum eigenvalues of sample covariance matrix respectively. The ratio between maximum and minimum eigenvalues $\lambda_{\max}/\lambda_{\min}$ is used to be the test statistic. We will compare MME detector with LEM detector from several aspects.

Receiver operating characteristic (ROC) curve is an essential graphical plot that illustrates the performance of a binary classifier system. Fig. 5 shows the ROC's comparison between MME detector and LEM detector and the latter has a better performance obviously compared with the former. Generally speaking, the spectrum sensing algorithm need to achieve constant false alarm rate (CFAR) detection and according to 802.22 working group, the target false-alarm probability in the CR is required to be less than 10%. As shown in the figure, when the false-alarm probability is 1%, the detection probability for MME detector and LEM detector is 64% and 92% respectively. Because it is unnecessary to estimate the noise power to obtain the threshold, both the MME detector and LEM detector do not suffer from a noise uncertainty problem which is the main disadvantage of the energy detector.

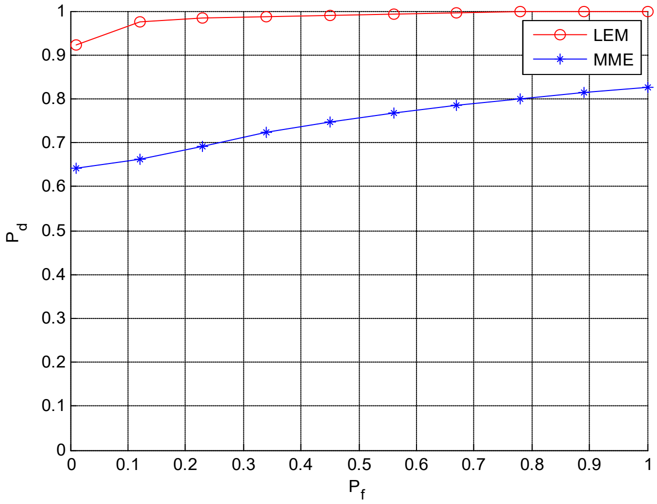


Fig. 5. ROC for MME and LEM

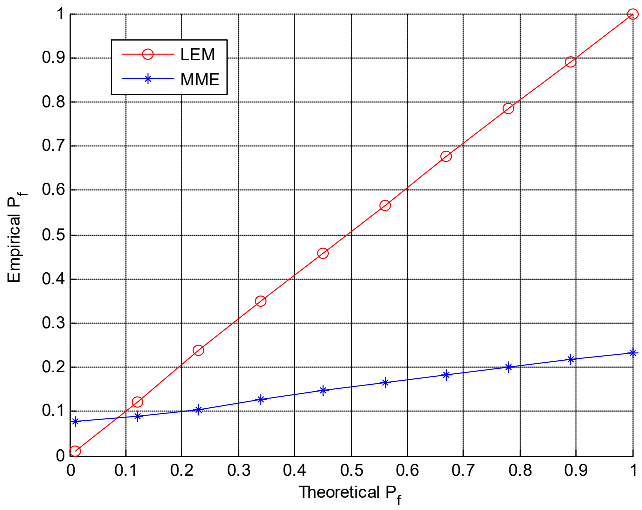


Fig. 6. Empirical P_f vs. theoretical P_f for MME and LEM

The abscissa coordinate of Fig. 6 is the theoretical false-alarm probability and the vertical coordinate is the empirical false-alarm probability. It shows that the curve of LEM algorithm is roughly diagonal, that is, the theoretical false-alarm probability is approximately equal to the theoretical false-alarm probability, which proves the correctness of the decision threshold derivation. As for MME algorithm, the empirical false-alarm probability deviates from the theoretical false-alarm probability significantly. MME detector is based on the asymptotic random matrix theory that requires the dimension of the matrix is infinite and it is impossible in practice, which leads to the deviation of the threshold.

C. *Comparison of performance under different SNR*

Fig. 7 represents that the detection probability for MME detector decreases sharply with the decrease of SNR while the detection probability for LEM detector decreases slowly. When the SNR is -20dB, the detection probability for MME detector and LEM detector is 65% and 95% respectively.

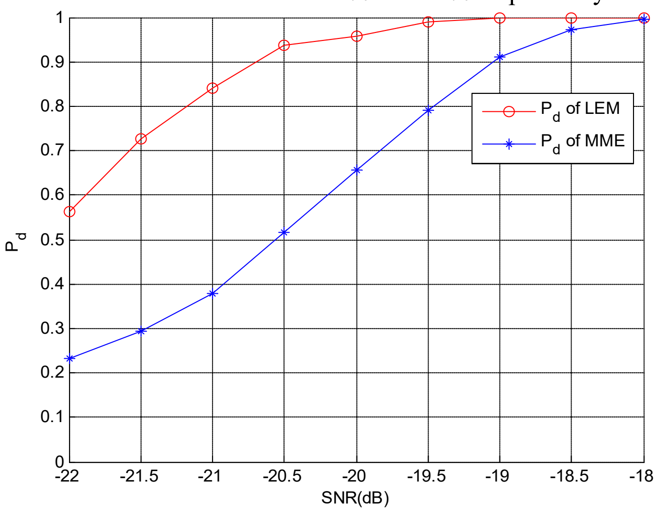


Fig. 7. P_d vs. SNR at $P_f = 5\%$ for MME and LEM

In Fig. 8, we investigate the probability of false alarm versus SNR. The false-alarm probability for LEM detector reaches 5% and fluctuates slightly around it. The false-alarm probability for MME detector is slightly higher than that for LEM algorithm because the threshold derivation of LEM algorithm is more accurate.

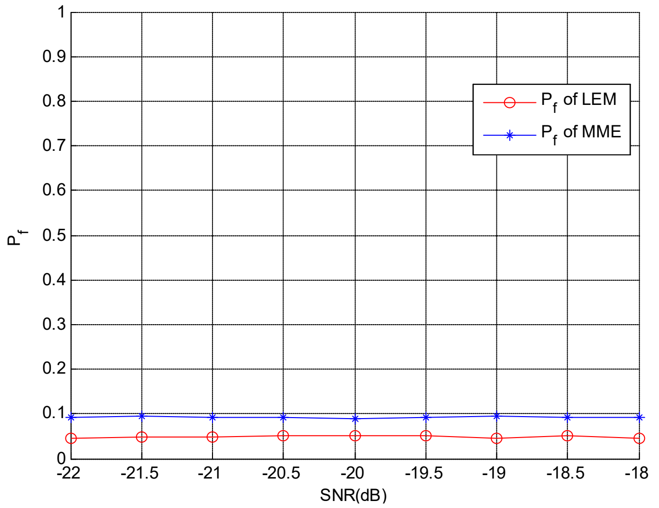


Fig. 8. P_f vs. SNR at $P_f = 5\%$ for MME and LEM

D. *The impact of the correlation between sensing nodes*

Reference [3] points out that MME detector requires the signal of sensing nodes are highly correlated, otherwise the detection probability falls down quickly. Due to its simplicity and flexibility, exponential model is widely adopted to describe correlation. ρ is the correlation coefficient between two sensing nodes that is related to the angular spread, wavelength and the distance between two nodes. Fig. 9 represents the detection probability for MME detector and LEM detector under different correlation coefficient. It is shown that the detection probability of MME approximates to zero as $\rho = 0.95$ while the probability of detection for LEM decreases a little in the same situation, which means the LEM algorithm is more robust against the decrease of correlation among the sensing nodes.

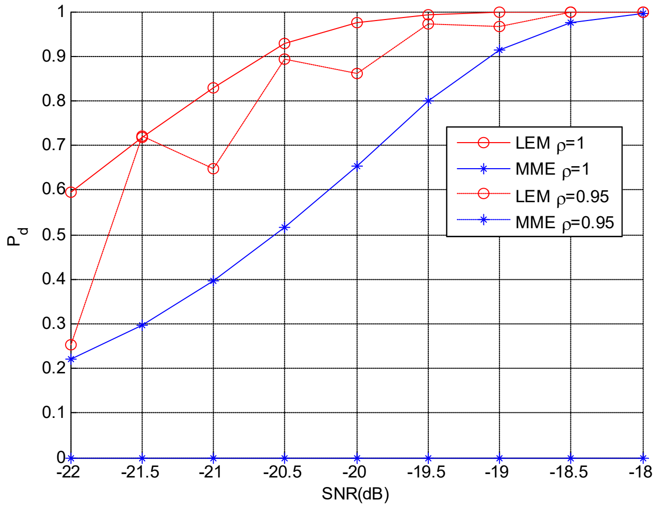


Fig. 9. P_d vs. SNR at $P_f = 5\%$ with different correlation

E. *The impact of the parameter K and N*

The detection probability for LEM detector with various numbers of CUs *K* and samples *N* is shown in Fig. 7 and Fig. 8 respectively. The simulation results show that the number of cooperative CUs *K* and the sample size *N* play the similar role in detection performance. It is obvious that the detection probability varies with the number of sensing nodes *K* proportionally. In addition, the probability of detection increases as the number of samples *N* grows.

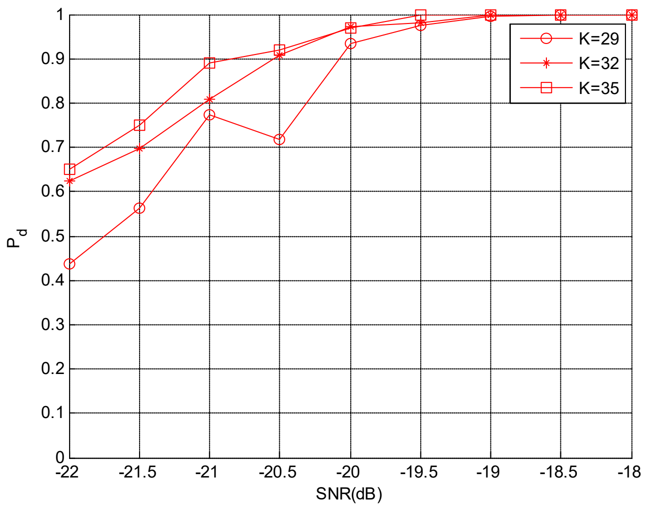


Fig. 10. P_d vs. SNR at $P_f = 5\%$ for LEM with different *K*

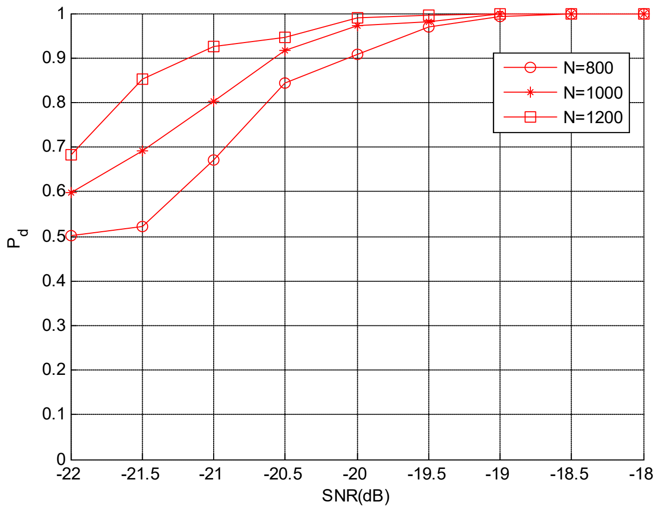


Fig. 11. P_d vs. SNR at P_f = 5% for LEM with different N

V. Conclusions

In this paper, a cooperative spectrum sensing algorithm using leading eigenvector matching is introduced. While PU's signal does not exist, the leading eigenvector is random. But when the signal is present, the leading eigenvector is stable. Due to its robustness, the feature can be learned blindly by FLA and LEM detector uses the feature as prior knowledge. The correlation coefficient between feature learned and the leading eigenvector of sample covariance matrix serves as the test statistic for signal detection. The closed-form expression of the threshold is also derived in this paper. Simulation results show that the algorithm proposed is reasonable and LEM detector outperforms MME detector. It also do not suffer from a noise power uncertainty problem. Compared with MME detector, LEM detector is more robust against the decrease of correlation among the sensing nodes. However there are some inherent flaws in this approach. A feature can only be learned in the presence of the desired PU signal, it cannot be learned in the presence of noise or in the presence of any other signal.

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