Channel Modeling and Analysis of ULA Massive MIMO Systems

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Abstract—In massive multiple-input multiple-output (MIMO) systems, the favorable propagation which can be also known as the orthogonality of the channel between different users is the most important property. In this paper, we establish a 3-D geometrical channel model for uniform linear array (ULA) massive MIMO systems, and we focus on the orthogonality of the channel, the condition number and the channel capacity. We use the plane wave (PW) to model the far-field signals, and use the spherical wave (SW) to model the near-field signals. Both the azimuth angle of arrival (AAoA) and elevation angle of arrival (EAAoA) have been taken into account. Compared with the independent and identically distributed (i.i.d.) Rayleigh fading channels, the proposed channel model comprises the parameters of communication environment and antennas such as environment scattering status, SW effect, antenna spacing etc. The relationship between the performance of massive MIMO systems and parameters is analyzed. The proposed model is easy to be implemented and can be adjusted according to the communication environment.

Keywords—Massive MIMO; favorable propagation; condition number; spherical wave.

I. INTRODUCTION

In recent years, the massive multiple-input multiple-output (MIMO) systems have been regarded as a candidate technology for the 5th generation (5G) cellular networks because they can achieve very high data rate and high energy efficiency [1]. Massive MIMO systems have a base station (BS) equipped with a large number of antennas (tens or hundreds of antennas) that are serving several single antenna users simultaneously. In massive MIMO systems, the favorable propagation which can be also called the orthogonality of the channel between different users is the most important property. A low channel orthogonality allows the BS to simultaneously serve different users with little cross-talk [2], furthering improving the channel capacity of massive MIMO systems. So far, there are a lot of works about channel orthogonality and capacity in developing massive MIMO systems. Hoyeris et al. [2] described a developed test-bed for outdoor channel measurements with very large antenna arrays and discussed the orthogonality of the channel vectors at different user positions. Gauger et al. [3] evaluated the cross-correlation between channel vectors over more than 400 terminal positions to quantify pairwise orthogonality. Gao et al. [4] also evaluated properties of measured channels with the BS equipped with 128 antenna ports and found that channel orthogonality was improved as the number of antennas increases.

Many works focus on the massive MIMO systems channel modeling because a channel model not only can reflect the propagation characteristic of signals and but also evaluate the performance of wireless communication systems without measurements [5]. Most of theoretical investigations [2] [5] [6] [7] are based on idealized channel matrix assumption using independent and identically distributed (i.i.d.) Rayleigh fading channels, which lacks parameters of communication environment and antennas. In addition, as the number of antennas increases, the plane wave (PW) condition is not met because the distance between the users and BS antennas is less than the Rayleigh distance where the huge size of antenna results in the great Rayleigh distance [8]. Therefore, the far-field assumption is no longer applicable to massive MIMO systems especially the uniform linear array (ULA) massive MIMO systems, and it is necessary to consider the spherical wave (SW) in near-field.

In this paper, we establish a 3-D geometrical channel model for ULA massive MIMO systems. ULA antennas configuration with equal antenna spacing is most common and widely used in real massive MIMO systems measurement campaigns [3] [9] [10]. We focus on the channel orthogonality, condition number and channel capacity. We use the PW to model the far-field signals, and the SW to model the near-field signals. Both the azimuth angle of arrival (AAoA) and elevation angle of arrival (EAAoA) have been considered. Compared with the i.i.d. Rayleigh fading channels, the proposed model has parameters of communication environment and antennas such as environment scattering status, SW effect, antenna spacing and so on. The relationship between the performance of massive MIMO systems and parameters is analyzed. The proposed model is easy to be implemented and can be adjusted according to the communication environment.

The remainder of this paper is organized as follows. Section II introduces the massive MIMO systems. Section III describes the channel model of ULA massive MIMO systems using both PW and SW. Section IV gives the simulation results.
The channel orthogonality also can be expressed as frequency resource at the same time [3]. As a consequence of completely correlated, and the two users cannot use the same antenna users are at random positions in the same cell. The antennas of the BS at user terminal position $M$ where $h_k$ denotes the Hermitian transform and $H$ is the channel vector using $M$ antennas of the BS at user terminal position $k$, and $h_{m,k}$ is the complex channel gain between BS antenna $m$ and user terminal position $k$. If the distance between the users and the BS antennas is far apart enough (i.e., longer than the Rayleigh distance), we can use the PW to model the channel. But when the distance between the users and BS antennas is less than the Rayleigh distance, we have to consider the SW to model the channel. For massive MIMO systems, the favorable propagation characteristic which can be also called channel orthogonality between different users is the most important property; namely [5],

$$\frac{1}{M} h_p^H h_q \xrightarrow{a.s.} M \rightarrow \infty \begin{cases} 0, & p \neq q \\ 1, & p = q \end{cases}$$  

(2)

where $H$ denotes the Hermitian transform and $\xrightarrow{a.s.}$ is almost sure convergence. For $\frac{1}{\sqrt{M}} h_p^H h_q \xrightarrow{a.s.} M \rightarrow \infty 0(p \neq q)$, it means that two users at positions $p, q$ are orthogonal to each other and there is little or even no cross-talk between them. While $\frac{1}{\sqrt{M}} h_p^H h_q \xrightarrow{a.s.} M \rightarrow \infty 1(p \neq q)$ means the two users are strongly or completely correlated, and the two users cannot use the same frequency resource at the same time [3]. As a consequence of (2), the channel orthogonality also can be expressed as

$$\frac{H^H H}{M} \xrightarrow{a.s.} M \rightarrow \infty I_K.$$  

(3)

In addition to the channel orthogonality, another indicator to measure whether the channels offer favorable propagation is condition number, which is defined as the ratio between the largest eigenvalue value $\sigma_{\text{max}}$ and the smallest eigenvalue value $\sigma_{\text{min}}$ of $H^H H$ [7]

$$\kappa = \frac{\sigma_{\text{max}}}{\sigma_{\text{min}}}.$$  

(4)

A large condition number means that the columns of the channel matrix are strongly correlated while $\kappa = 1$ means that all columns are orthogonal and the channels offer favorable propagation. The favorable propagation can reduce the cross-talk between users and simplify the precoding of massive MIMO systems, which is beneficial to the channel capacity. With the power allocation, the capacity for massive MIMO systems is given as [9]

$$C = \max_{\mathbf{P}} \log_2 \left[ \det (\mathbf{I} + \frac{\beta K}{M} \mathbf{H P H}^H) \right] \text{bps/Hz}$$  

(5)

where $\mathbf{I}$ is the identity matrix, and $\beta$ denotes the receiving signal-to-noise ratio (SNR). $\mathbf{P}$ is a diagonal matrix for power allocation with $(p_1, p_2, \ldots, p_K)$ on its diagonal and $\sum_{k=1}^{K} p_k = 1$.

III. 3-D CHANNEL MODELING OF ULA MASSIVE MIMO SYSTEMS

A. Channel Modeling Using the PW

A 3-D ULA massive MIMO system transmission scenario using PW is shown in Fig. 2. All the antennas are in the far-field of the signals, and the signals at antennas are parallel, which only have transmission distance differences. The $x$-$S_1$-$y$ plane is the horizontal plane. The signals come from arbitrary direction with AAoA $\alpha$ and EAoA $\beta$ for all the antennas. The adjacent antenna spacing is $d$. For antenna $A_2$, $S_2 P_2'$ is the projection of $S_2 P_2$ on the $x$-$S_1$-$y$ plane. $\angle P_2 S_2 P_2'$ is $\beta$, and $\angle P_2 S_2 S_1$ equals to $(\pi/2 - \alpha)$. Therefore, the angle $\angle P_2 S_2 S_1$ is given by

$$\cos \angle P_2 S_2 S_1 = \cos \angle P_2 S_2 S_1 (\cos \angle P_2 S_2 P_2')$$

$$= \cos (\pi/2 - \alpha) \cos (\beta)$$

(6)

where (6) is proved in [11]. The transmission distance difference between antennas $A_1$ and $A_2$ is $P_2 S_2$. Regarding antenna
\( h_1^{PW} = A e^{j \phi} \) \hspace{1cm} (7)
\( h_2^{PW} = A e^{j(\phi + 2 \pi d \cos(\frac{x}{\lambda} - \alpha) \cos(\beta)/\lambda)} \) \hspace{1cm} (8)

where \( A \) is the receiving amplitude, \( \phi \) is the receiving phase and \( \phi \) is i.i.d. uniform random variable on the interval \([-\pi, \pi] \). \( \lambda \) is the carrier wavelength. In ULA massive MIMO systems, all the antennas are uniformly-spaced \([8] [9]\), therefore, for antenna \( A_m \), one path of multipath channels can be expressed as

\[ h_m^{PW} = A e^{j(\phi + 2 \pi d (m-1) \cos(\frac{x}{\lambda} - \alpha) \cos(\beta)/\lambda)}. \] \hspace{1cm} (9)

Therefore, for user position \( p \), the channel vector using the PW can be written as

\[ h_p^{PW} = \begin{bmatrix} h_{1,p}^{PW} \\ \vdots \\ h_{M,p}^{PW} \end{bmatrix} = \begin{bmatrix} A e^{j \phi} \\ \vdots \\ A e^{j(\phi + 2 \pi d (m-1) \cos(\frac{x}{\lambda} - \alpha) \cos(\beta)/\lambda)} \end{bmatrix}. \] \hspace{1cm} (10)

For multipath channels, there are many different distributions to characterize the AAoA/E AoA distributions. Here we use the uniform distribution with certain azimuth angle spread (AAS) and elevation angle spread (EAS) to characterize the AAoA/E AoA distributions because many communication scenarios are consistent with uniform distribution \([13]\), which is defined as

\[ p(\theta) = \frac{1}{2\Delta \theta}, -\Delta \theta + \theta_0 \leq \theta \leq \Delta \theta + \theta_0 \] \hspace{1cm} (11)

where \( \theta_0 \) is the mean AAoA/E AoA, and \( \Delta \theta \) is the AAS/EAS.

In the same way, the channel vector at user position \( q \) using PW is

\[ h_q^{PW} = \begin{bmatrix} h_{1,q}^{PW} \\ \vdots \\ h_{M,q}^{PW} \end{bmatrix} = \begin{bmatrix} A e^{j \phi_q} \\ \vdots \\ A e^{j(\phi_q + 2 \pi d (m-1) \cos(\frac{x}{\lambda} - \alpha) \cos(\beta)/\lambda)} \end{bmatrix}. \] \hspace{1cm} (12)

Therefore, the complete channel matrix using the PW is

\[ H^{PW} = [h_1^{PW} h_2^{PW} \ldots h_k^{PW} \ldots h_M^{PW}]. \] \hspace{1cm} (13)

We are interested in the favorable propagation characteristic of massive MIMO systems. After getting the complete channel matrix, we can derive the orthogonality between two channel vectors \( h_p^{PW} \) and \( h_q^{PW} \), which is computed as \([6]\]

\[ \delta_{p,q}^{PW} = \frac{||h_p^{PW}||}{||h_q^{PW}||} \] \hspace{1cm} (14)

where \( || \cdot || \) denotes the Euclidean norm. Also we can get the condition number and the channel capacity.

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**B. Channel Modeling Using the SW**

As mentioned earlier, when the distance between the users and BS antennas is less than the Rayleigh distance, the PW condition is dissatisfied, and then we have to consider the SW. Fig. 3 shows the 3-D ULA massive MIMO systems transmission scenario using the SW. From Fig. 3 we can see that the signals at the antennas are sphere, which are not parallel anymore. The signals come from source \( S \), and \( S' \) is the projection of \( S \) on the horizontal plane \( x-S_1-y \). The distance between the \( S' \) and the \( x \) axis is \( d_x \), the distance between the \( S' \) and the \( y \) axis is \( d_y \), and the height of \( S \) is \( h \). Then we can obtain the exact distances between the source \( S \) and every antenna

\[ d_{SS_1} = \sqrt{d_x^2 + d_y^2 + h^2} \]
\[ d_{SS_2} = \sqrt{(d_x + d)^2 + d_y^2 + h^2} \]
\[ \ldots \]
\[ d_{SS_M} = \sqrt{(d_x + (M-1)d)^2 + d_y^2 + h^2} \] \hspace{1cm} (15)

Also we regard antenna \( A_1 \) as a reference antenna, and the channels of different antennas using the SW are

\[ h_1^{SW} = A e^{j(\phi + 2 \pi \sqrt{d_x^2 + d_y^2 + h^2}/\lambda)} \]
\[ h_2^{SW} = A e^{j(\phi + 2 \pi \sqrt{(d+x)^2 + d_y^2 + h^2}/\lambda)} \]
\[ \ldots \]
\[ h_m^{SW} = A e^{j(\phi + 2 \pi \sqrt{(d+(m-1)d)^2 + d_y^2 + h^2}/\lambda)} \] \hspace{1cm} (16)

According to the geometrical relationship in Fig. 3, for an arbitrary antenna we have

\[ d_x = \tan(\alpha^m) \cdot d_y - d(m-1) \]
\[ h = \tan(\beta^m) \sqrt{\tan^2(\alpha^m) \cdot d_x^2 + d_y^2}. \] \hspace{1cm} (17)
Therefore, the channel vector at user position \( p \) and \( q \) using the SW are

\[
\mathbf{h}_{p q}^{SW} = \begin{bmatrix}
    h_{1 q}^{SW} \\
    h_{2 q}^{SW} \\
    \vdots \\
    h_{M q}^{SW}
\end{bmatrix} = \begin{bmatrix}
    Ae^{j(\theta_1 + 2\pi \sqrt{d_{1q}^2 + d_{1q}^2 + h_{1q}^2/\lambda})} \\
    Ae^{j(\theta_2 + 2\pi \sqrt{(d + d_{1q})^2 + d_{1q}^2 + h_{1q}^2/\lambda})} \\
    \vdots \\
    Ae^{j(\theta_{Mq} + 2\pi \sqrt{(d(M-1) + d_{1q})^2 + d_{1q}^2 + h_{1q}^2/\lambda})}
\end{bmatrix}.
\]

The channel matrix using the SW is

\[
\mathbf{H}^{SW} = [\mathbf{h}_1^{SW} \mathbf{h}_2^{SW} \cdots \mathbf{h}_q^{SW} \cdots \mathbf{h}_M^{SW}].
\]

Then we can also get the channel orthogonality, condition number and the channel capacity using SW as PW.

IV. SIMULATION RESULTS AND ANALYSIS

From Fig. 4(a) we can see the channel orthogonality using the PW varies as the number of antennas increases with different AAS and EAS. The antenna spacing is equal to \( \lambda/2 \), both the mean AAoA and mean EAoA are set to be 0\(^\circ\) and the amplitude is normalized. We use the Monte Carlo simulation method, where we generate 10000 samples of the channel and compute the average channel orthogonality. We observe that the channel orthogonality decreases as the number of antennas increases. The channel orthogonality is sensitive to the AAS while the EAS has a slight effect on the orthogonality since it is placed in the horizontal plane, and a larger AAS results in a lower orthogonality. When the number of antennas is 20, the channel orthogonality with AAS=3\(^\circ\) is nearly 0.75, while the channel orthogonality with AAS=30\(^\circ\) is lower than 0.2. When the number of antennas becomes 200, the channel orthogonality with AAS=3\(^\circ\) is also lower than 0.2, which means as the number of antennas increases, the channel orthogonality with small AAS can also decline to a small value. Therefore, a large number of antennas at BS can help to decline the channel orthogonality between users, especially under a poor scattering communication environment, and massive antennas can reduce the demand for rich scattering compared with the traditional MIMO.

In addition to the scattering environment, the channel orthogonality is also sensitive to the antenna spacing. Fig. 4(b) draws the channel orthogonality using the PW varies as antenna spacing changes. AAS is set to be 10\(^\circ\) and EAS is set to be 30\(^\circ\). From which we can see that the larger antenna spacing results in a lower channel orthogonality. Also, when the number of antennas is small, the large antenna spacing has great advantage, while the channel orthogonality declines to a small value as the number of antennas increases even when the antenna spacing is small. Therefore, massive antennas can reduce the demand for large antenna spacing compared with the traditional MIMO, and usually the adjacent antenna spacing in massive MIMO systems is \( \lambda/2 \) at the BS. In addition, from both Fig. 4(a) and Fig. 4(b), we can see that adding antennas is always beneficial to the channel orthogonality, but the channel orthogonality decreases to a certain extent value as the number of antennas increases, and to continue increasing the number of antennas makes very little improvement. Similar effects are also observed in [2] [4] and [14] under real propagation environments.

Fig. 5 shows the channel orthogonality comparison of using PW and SW. From Fig. 5 we can see that the SW
Fig. 5. Channel orthogonality comparison of using PW and SW versus number of antennas.

curves overlap the PW curve when the number of antennas is small (less than 20). As the number of antennas increases, the channel orthogonality using the PW and SW becomes different, and channel orthogonality using the SW is a little higher than using the PW. This is because when the number of antennas is small (i.e., the size of the massive antenna structure is small), the distance between the users and the BS antennas is beyond the Rayleigh distance and the antennas are in the far-field, hence the SW effect is weak or the SW can be regarded as PW. As the number of antennas increases to a huge size, the antennas are in the near-field and the SW effect becomes obvious. Also we can see as the distance between the users and the BS antennas becomes larger, the SW curve gets closer to the PW curve, that is because the SW effect is weakened as the distance between the users and the BS antennas becomes larger.

As discussed earlier, condition number is another indicator to measure whether the channels offer favorable propagation. In order to obtain a finite range picture of condition number, we consider the inverse of condition number $\kappa^{-1} \in [0, 1]$ as shown in Fig. 6(a). The condition number is improved as the number of antennas increases, and the SW has the poorer condition number because of the higher channel orthogonality. Also, large AAS and large antenna spacing can result in the better condition number. Finally, Fig. 6(b) shows the channel capacity using both PW and SW with different parameters setting. SNR is set to be 20 dB, $K = 8$ and the power is equally allocated. The channel capacity increases as the number of antennas increases, and it seems that the channel capacity is the inverse of the channel orthogonality. We can see that a larger AAS results in a larger channel capacity because a larger AAS makes a lower channel orthogonality. Also, the channel capacity is influenced by the antenna spacing. A larger antenna spacing can achieve larger channel capacity because of the lower channel orthogonality. For the poor scattering communication environment (small AAS), increasing the antenna spacing can make significant improvement in the channel capacity, while when AAS is already large, increasing the antenna spacing makes little improvement in channel capacity because the channel orthogonality is already low. Therefore, when the scattering is rich in the communication environment, we can make antennas more compact to realize the space efficiency. While if the scattering is poor in the communication environment, we can increase the antenna spacing to enhance the systems performance. The channel capacity difference between the PW and SW becomes obvious as the number of antennas increases, and the channel capacity using PW is a little higher than using the SW because of the lower channel orthogonality. Also, adding antennas is always beneficial to the channel capacity, while it seems that 60 antennas at BS is enough for achieving high data rate in different communication
environment and there is very little improvement beyond 60 antennas.

V. CONCLUSION

In this paper, we propose a 3-D geometrical channel model for ULA massive MIMO systems. The proposed model has the information of communication environment and antennas, and it can be adjusted according to the communication environment. The simulation results show that the channel orthogonality is sensitive to the AAS and antenna spacing, and it decreases as the number of antennas increases. The channel orthogonality using the SW is a little higher than using the PW, and the SW effect becomes obvious as the number of antennas increases. Adding antennas is always beneficial to the systems, but the system performance increases to a certain extent as the number of antennas increases, and to continue increasing the number of antennas makes very little improvement.

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