Frequency Domain Estimation and Time Domain Correction of CFO and STO Offsets in OFDM System

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Abstract— **OFDM can use frequency band efficiently and has strong characteristic for facing channel. But orthogonality collapse when carrier frequency offset and sample timing offset occur. Sample timing offset is represented as phase rotation and ISI. And Carrier frequency offset is represented as phase rotation and ICI. To compensate those effects, many studies were processed based on frequency domain. In this paper, we estimate those effects in frequency domain. And the effects compensate in time domain by using feedback method.**

Keywords— **Carrier Frequency Offset, Inter-carrier Interference, OFDM, Sample Timing Offset, Feedback**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a method that transmits subcarriers as parallel. OFDM has strong characteristic of impulse noise and can use frequency band efficiently. So OFDM is used in wireless communication standard such as wireless LAN, digital video broadcasting, digital audio broadcasting system. But Doppler or mismatch between transmitter and receiver make carrier frequency offset (CFO) [1-2] and mismatch between DAC and ADC make sample timing offset (STO) [3].

CFO is represented shift in frequency domain, and orthogonality collapse. At that time, phase rotation and inter-carrier interference (ICI) occur [4]. Phase rotation is compensated by pilot but ICI need complex equalizer. STO is represented phase rotation and inter-symbol interference (ISI). ISI is removed using cycle prefix (CP). But phase error that is generated per subcarriers degraded BER performance. To fine out synchronization, there were many studies. One method is to use preamble [5]. And also scatter type pilot [6] and comb type pilot are used for compensation.

In this paper, we analyze joint environment of CFO and STO in OFDM system. Especially, when sample delay is smaller than one sample distance, we analyze that the effect is how to be represented. And we estimate the effect using pilot and correct it after feedback. We can correct CFO to control frequency of fractional-PLL and STO to control sample timing of Sample & Holder.

The rest of this paper is organized as follows. Section II presents the OFDM system model that has feedback structure. In Section III, we analyse the effects of CFO, STO. Section IV estimate the effects and propose how to compensate that effects. Simulation results and discussion are presented in Section V. Conclusions are drawn in Section VI.

II. OFDM SYSTEM MODEL

Assumed signal with N sub-carrier is transmitted as a signal with N samples after IFFT.

$$x(k)=\sum_{n=0}^{N-1} D\cdot e^{i2\pi\frac{n}{N}} \tag{1}$$

A system structure is (1), which has feedback method. In this system, we assume CFO is generated due to mismatch between transmitter and receiver oscillator and Doppler and STO occur because of timing mismatch between DAC and ADC. So CFO and STO are generated at RX Front-end and Sampling & Holder.

We propose a method that estimate CFO and fractional type STO in frequency domain, and correct them in time domain by doing feedback.

III. OFFSET ANALYSIS

A. Carrier frequency offset

Output signal of TX front-end is continuous signal. To obtain good performance, the receiver gets exact sample timing. However, oscillators between transmitter and receiver are not same exactly and Doppler is generated due to mobility of receiver. Especially, CFO destroys orthogonality of OFDM and makes ICI. And it is represented performance degradation.

Data x(t) is represented like (2) because of CFO.

$$y_n=\sum_{k=0}^{N-1} H_k\cdot X_k\cdot e^{i2\pi\frac{k+\varepsilon}{N}}+z_n \tag{2}$$

CFO is represented as phase rotation in time domain. The effect makes phase error and ICI in frequency domain.

$$\begin{aligned} Y_p&=\sum_{m=0}^{N-1}\sum_{k=0}^{N-1} H_{k,m}\cdot X_{k,m}\cdot e^{i2\pi\frac{(k+\varepsilon)}{N}}\cdot e^{-i2\pi\frac{m}{N}}+Z_p \\ &=H_p\cdot X_p\, e^{i2\pi\varepsilon p}+\sum_{\substack{m=0\\ m\neq k}}^{N-1}\sum_{k=0}^{N-1} H_{k,m}\cdot X_{k,m}\cdot e^{i2\pi\frac{(k-m)}{N}}\cdot e^{i2\pi\frac{\varepsilon}{N}}+Z_p \end{aligned} \tag{3}$$

If sampled discrete signal is distorted by CFO, phase rotation and ICI is generated like (3). Zp is sampled AWGN. Phase error makes more performance degradation than ICI, but it is compensated by pilot easily.

ε is normalized carrier frequency offset. In other words, it is value that divide Doppler frequency into carrier spacing. We assume receiver move to transmitter direction.

$$\varepsilon = \frac{f_d}{carrier\ spacing}\ ,\ f_d = \frac{v \cdot f_c}{c} \tag{4}$$

B. Sample timing offset

Sample timing offset is considered as two types. One is sample unit STO. In other words, delay distance is sample distance. The other is that delay distance is smaller than one sample. We consider two types of STO.

First is integer type STO. A signal in OFDM system is sampled as a signal with N samples after IFFT and the signal is processed one symbol. If a symbol is delayed, the symbol affect next symbol. Therefore, ISI is occurs. ISI can be prevented using cycle prefix (CP), so all symbols maintain orthogonality. But phase rotation remains. Integer type STO makes larger rotation than fractional type STO. Equation (5) represents phase rotation and ISI.

$$y_n = h_n \otimes x_{n+\delta} + z_n$$

$$Y_p = \sum_{m=0}^{N-1} \sum_{k=0}^{N-1} H_{k,m} \cdot X_{k,m} \cdot e^{i2\pi\frac{k-m}{N}} \cdot e^{i2\pi\frac{\delta m}{N}} + Z_p$$
$$= H_p \cdot X_p \sum_{m=0}^{N-1} e^{i2\pi\frac{-\delta m}{N}} + \sum_{\substack{m=0 \\ m\neq k}}^{N-1} \sum_{k=0}^{N-1} H_{k,m} \cdot X_{k,m} \cdot e^{i2\pi\frac{k-m}{N}} \cdot e^{i2\pi\frac{(\delta m)}{N}} + Z_p \tag{5}$$

Integer type STO makes phase rotation per subcarrier due to OFDM orthogonal characteristic. That constellation is circled. But Integer type STO has minimum sampling rate at low and high position of subcarrier. So one sample delay make phase rotation of 180 degree. If delay is increased, it make difficult to estimate the effect. Therefore block type pilot is used.

Second is fractional type STO. A continuous signal is sampled a discrete signal through ADC. It is possible that smaller delay is generated than one sample. Fractional type STO has less phase rotation than integer type STO. But if fractional type STO is increased, BER performance is decreased gradually. And compensation also is difficult because the signal is sampled. Fractional type STO has similar equation like integer type STO.

$$Y_p = \sum_{m=0}^{N-1} \sum_{k=0}^{N-1} H_{k,m} \cdot X_{k,m} \cdot e^{i2\pi\frac{k-m}{N}} \cdot e^{i2\pi\frac{\Delta\delta m}{N}} + Z_p$$
$$= H_p \cdot X_p \sum_{m=0}^{N-1} e^{i2\pi\frac{-\Delta\delta m}{N}} + \sum_{\substack{m=0 \\ m\neq k}}^{N-1} \sum_{k=0}^{N-1} H_{k,m} \cdot X_{k,m} \cdot e^{i2\pi\frac{k-m}{N}} \cdot e^{i2\pi\frac{\Delta\delta m}{N}} + Z_p \tag{6}$$

When the first subcarrier has one cycle, next subcarrier has two cycles in OFDM. Because of these characteristic, STO is represented as phase rotation per subcarrier in OFDM system.

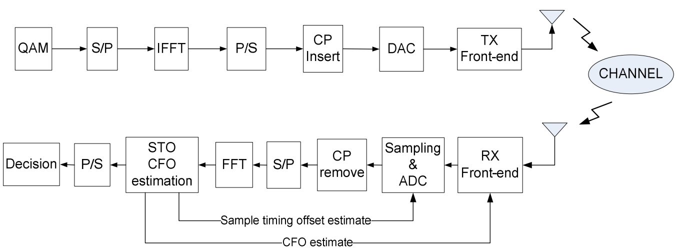

Figure 1. Proposed OFDM system model.

Fractional type STO has similar effect like integer type STO. BER performance degradation of fractional type STO is smaller than integer type STO. But it also make difficult to receive correct data due to phase rotation. And if minimum sampled signals have time delay about 90 degree, BER performance is degraded. In this case, compensation using equalizer is to be difficult.

IV. CFO and STO Estimation

A. CFO estimation

CFO estimation is done by block pilot to fine phase rotation degree. Equation (8) represents phase error due to CFO.

$$Y_p = H_p \cdot X_p\ e^{i2\pi\varepsilon p} + Z_p \tag{8}$$

We suppose channel is AWGN. And we can estimate error using pilot. First, we can fine out phase error at the received signal. CFO's phase rotation is generated in the same direction. So estimation values of CFO can search like (9)(10).

$$P_{received} = P_{transmitted} \cdot e^{i2\pi\varepsilon} \tag{9}$$

Equation (9) is represented phase error without ICI.

$$\hat{\varepsilon} = \ln \frac{P_{received}}{P_{transmitted}} / \left(j2\pi \right) \tag{10}$$

And estimation value is show (10).

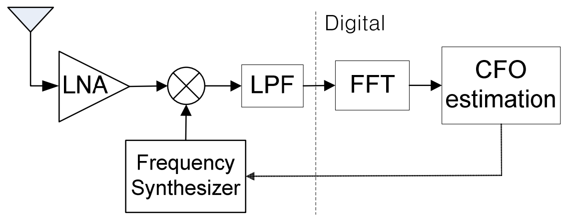

Figure 2. Correction of CFO using fractional PLL.

Figure 2 is block diagram of fractional PLL. Frequency of oscillator can control by moving frequency of fractional PLL. The frequency of fractional PLL is by decided estimation value that is gotten through (10). So the effect about offset will be minimized.

B. STO estimation

STO also has phase error and ISI. And STO estimate using pilot.

$$Y_p = H_p \cdot X_p \sum_{m=0}^{N-1} e^{-i2\pi\frac{(\delta + \Delta\delta)m}{N}} \tag{11}$$

We consider the effects that are integer type STO and fractional type STO like Equation (11). With CP, OFDM system has orthogonality. So STO make phase rotation. But difference of each subcarrier causes each phase rotation per subcarrier. Because Integer type STO has large phase rotation, integer type STO is estimated by using synchronization symbol like (12).

$$\hat{\delta} = \arg\min \left\{ \sum_{i=\delta}^{N_G - 1 + \delta} \left| Sync_{received}[n+i] - Sync^*_{pilot}[i] \right|^2 \right\} \tag{12}$$

Equation (12) is search minimum error using PN sequence.

$$Y_p = H_p \cdot X_p \sum_{m}^{N-1} e^{i2\pi\frac{-\Delta\delta\, m}{N}} + Z_p \tag{13}$$

After integer type STO is compensated by synchronization symbol, fractional type STO is estimated as phase rotation degree.

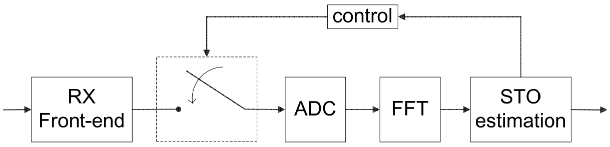

Figure 3. Correction of STO by controlling sample timing.

Figure 3 is block diagram of correction method of STO. The value that is estimated in frequency domain is corrected in time domain by controlling Sample & Holder. Like figure 3, received signal through RX Front-end is sampled with n period and then continuous signal is to be discrete signal. At that time the sampling time is changed. So a wide band ADC and fast switch is needed.

Figure 4 shows performance when fractional type STO is generated. As you can see at figure 4, when fractional type STO occur, we can see degraded BER performance from 0.05 sample delay. So we set minimum value as 0.05 for estimation.

When STO occur, a single after IFFT has different phase degree per subcarrier. If 0.1sample delay occurs, the rotated phase is from 0.5 to 18 degree. In frequency domain, subcarriers have same phase rotation degree between subcarrier number 1~32 and 33~64. But the two group of subcarrier have different phase direction. So it is represented like (14).

$$\frac{Y_p}{X_p} = \sum_{p=1}^{32} e^{\frac{-i2\pi\Delta\delta p}{N}}, \quad \frac{Y_p}{X_p} = \sum_{p=33}^{64} e^{\frac{-i2\pi\Delta\delta p}{N}} \tag{14}$$

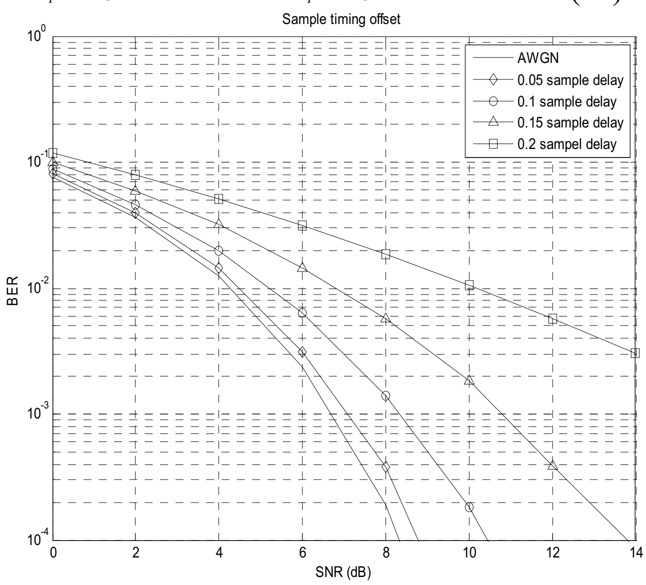

Figure 4. BER performance due to fractional type sample timing offset.

And it also is represented like (15).

$$\left| \sum_{p=1}^{32} e^{\frac{-i2\pi\Delta\delta p}{N}} \right| = \left| \sum_{p=33}^{64} e^{\frac{-i2\pi\Delta\delta p}{N}} \right| \tag{15}$$

So we can calculate fractional estimation value like (16). Here, P_i-$P_{i+N/2}$ is similar regardless of i. So estimated value is found using average of P_i-$P_{i+N/2}$.

$$\Delta\hat{\delta} = \frac{\sum_{i=1}^{N/2}(P_i - P_{i+N/2})}{N/2} \tag{16}$$

The minimum control time of Sample and Holder is set by figure 4, when estimation values do feedback.

V. Simulation Results and Discussion

Table 1. Simulation Parameters.

OFDM system	
Number of Subcarrier	64
Subcarrier Spacing	7.812kHz
Symbol Period	$160\mu s$
Sampling Frequency	500KHz
Sample Period	$2\mu s$
CP Length	16
Modulation	QPSK
Channel	AWGN

Table 1 shows simulation parameters. OFDM signal has 64 subcarriers and modulation is QPSK. Normalized CFO can calculate by substituting table 1 values at (4).

If carrier frequency is set as 3GHz, Doppler frequency is 333.33Hz. Therefore normalized CFO is 0.042 and receiver direction is toward the transmitter. At that time, Doppler frequency is maximum. We consider mismatch between transmitter and receiver as well as Doppler, so CFO is set as 0.05.

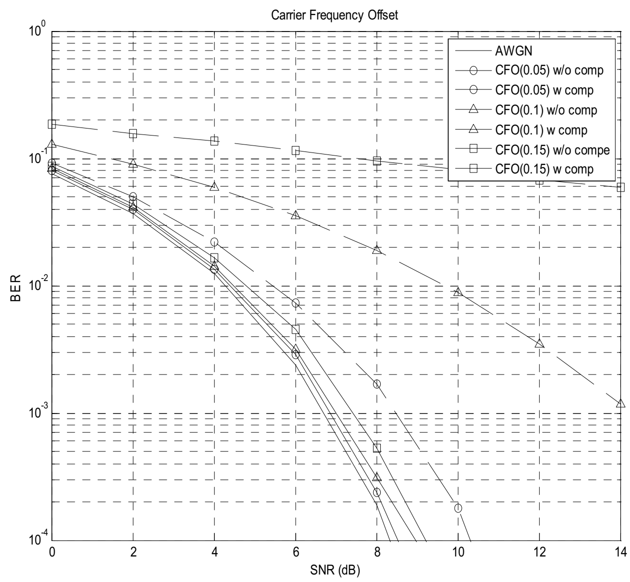

Figure 5. BER performance according to CFO changes.

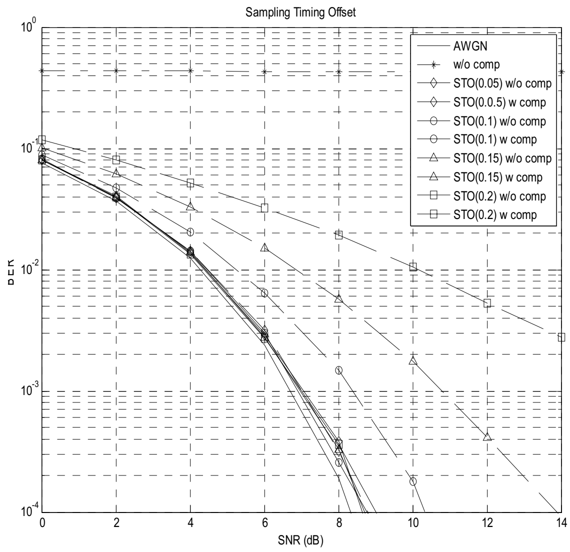

Figure 6. BER performance according to STO changes.

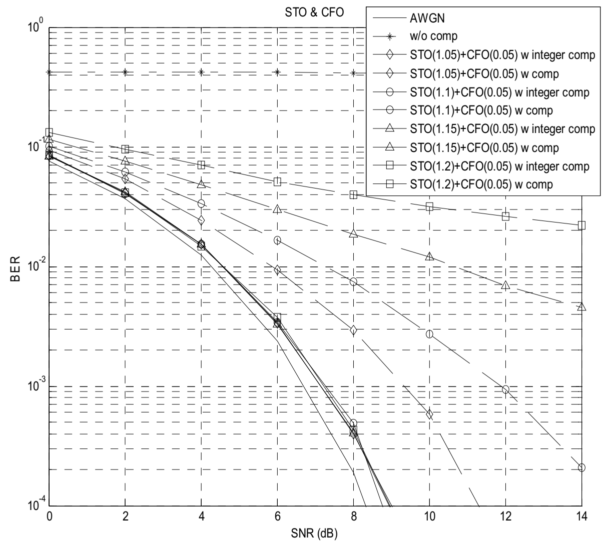

Figure 7. BER performance at joint environment with STO and CFO.

First of all, CFO cause phase error and ICI. Figure 5 show BER performances when CFO occurs. When CFO is 0.05, 0.1, 0.15, BER performances is degraded. But if frequency of oscillator can control using fractional PLL, performances is corrected like figure 5. When CFO is 0.05, SNR is about 10.5dB at BER 10e-4 without correction. But after correction, BER is about 8.5dB. As a simulation result, we can know feedback method using fractional PLL is good.

Figure 6 is BER performance of STO. STO just has phase rotation with CP. But phase degree is so large to receive accurate data. When integer type STO exists, we can't receive data. And after integer type STO is compensated, BER performances also are degraded because of fractional type STO. There are example values of fractional type STO such as 0.05, 0.1, 0.15, 0.2. If fractional type STO is increased gradually, BER performance is decreased. But after correction, we can get improved performances.

All corrected data has estimation error in figure 6, because AWGN and ICI cause estimation error. And if minimum sampled signal has delay, that also generate estimation error.

In figure 7, when integer type STO occurs, although SNR is increased, we can't communicate. And fractional type STO occurs, BER performance is decreased gradually according to delay size. Estimation error is about 0.05 samples, because we control Sample & Holder about 0.05 per sample. And smaller errors are hard to estimate than 0.05 samples.

Figure 7 is simulation at joint environment. Normalized CFO considering Doppler and mismatch between transmitter and receiver is set as 0.05. STO is set as 1.05, 1.1, 1.15, 1.2 sample delay.

In joint environment, though integer type STO is corrected, fractional type STO and CFO is to be reason about performance degradation. It can't also communicate. As a simulation result, after correction, BER performance is higher about 1dB than AWGN at 10e-4. The difference of BER performance is due to estimation error CFO and fractional type STO.

VI. CONCLUSION

We study CFO that is generated by Doppler and mismatch transmitter and receiver, and STO that mismatch between DAC and ADC. These Two effects are large reason of performance degradation in OFDM system.

In this paper, we study a method using feedback to correct STO and CFO. And we simulate those effects and proposed correction method. As a result simulation, we show the method using feedback, which also has good performance as much as use equalizer. Moreover, we can consider the fact that it can correct in time domain. This system is possible to reduce complexity of equalizer and price of receiver.

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