Betweenness Centralization Analysis Formalisms on Workflow-Supported Org-Social Networks

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Abstract—The purpose of this paper is to conceive an algorithmic approach to measure betweenness centralities among performers in a workflow-supported org-social network model. The essential part of the approach is a betweenness centrality analysis algorithm to calculate each performer’s betweenness centrality and group betweenness centrality on a corresponding workflow-supported org-social network model. We strongly expect that the developed algorithm will be applied to analyzing the degree of work-mediation of each of the performers who are allotted to perform a corresponding workflow procedure.

Keywords: workflow-supported org-social networking knowledge, ICN-based workflow model, betweenness centrality analysis, organizational knowledge discovery, business process intelligence

I. INTRODUCTION

Recently, the workflow literature just starts being focused on “People[1][2].” It starts from the strong belief that organizational relationships and collaborative behaviors among people who are involved in enacting the specific workflow procedures affect the overall performance and being crowned with great successes in the real businesses and the working productivity as well. So, research and development issues of amalgamating the concept of social networks with the workflow-supported organizational behaviors have arisen in the literature as the likes of “workflow-supported org-social network[3][4]” and “workflow-supported org-affiliation network[5][6].” There are two main branches of the issues in adopting the social network techniques[7] onto workflow-supported organizational behaviors; One is discovery issue, and the other is rediscovery issues. The latter is concerned with mining org-social networking knowledge from workflow enactment event logs, which was firstly issued by Aalst, et al.[8]; the former is to discover org-social networking knowledge through exploring the human perspective of a group of workflow models, which was issued at first by Song, et al. [3].

We are particularly interested in an algorithmic analysis and measurement approach of a workflow-supported org-social network. The authors’ research group has been developing a framework[3][9] based upon the basic concept of workflow-supported org-social networks and its related analysis methods, like centrality, prestige, and clique analysis equations and techniques[7]. It ought to be definitely necessary for the framework to be equipped with more sophisticated and diversified analysis techniques, such as degree-centrality, closeness-centrality, betweenness-centrality, eigenvalue-centrality, correspondence analysis, and so on, in order to be practically applied into a real organizational world. As one of those efforts, in this paper, we try to conceive an algorithmic formalism of betweenness-centrality measurements to quantitatively analyze workflow-supported org-social networking knowledge and models. The eventual goal of the formalism is to numerically measure and calculate the degree of work-mediation among employees involved in a corresponding workflow procedure on a workflow-supported organizational environment.

II. PRELIMINARY

In this section, we start from introducing the basic concept and definition of workflow-supported org-social network model that can be used for a knowledge representation theory of workflow-supported org-social networking knowledge that might be either discovered from workflow models or rediscovered from workflow execution logs. Basically, the origin of the workflow-supported org-social network model is the actor-based workflow model[1], and its rationale is on where it represents the behaviors of acquisition activities among actors in a workflow model, which we would call workflow-supported org-social relationships that form this special type of social networks.

As given in the formal definition, [Definition 1], of the workflow-supported org-social network model, the behaviors of the model are revealed through incoming and outgoing directed arcs labeled with activities associated with each of actors. The directed arcs imply two kinds of behaviors—workflow-supported social relationships and activity acquisition of actors—through which we are able to get precedence (candidate-predecessor knowledge/candidate-successor knowledge) knowledge among actors as well as activity acquisition of each actor in a workflow model. In terms of defining actor’s predecessors and successors, we would use the prepositional word, “candidate,” because a role-actor mapping is an one-to-many relationship knowledge, and the actor selection mechanism will choose one actor
out of the assigned actors mapped to the corresponding role during the underlying workflow model’s runtime.

[Definition 1] Workflow-supported Org-Social Network Model. A workflow-supported org-social network model is formally defined as $A = (\sigma, \psi, S, E)$, over a set $C$ of performers, and a set $A$ of activities. Note that $\varphi()$ is a power-set function.

- $S$ is a finite set of coordinators or coordinator-groups connected from some external workflow-supported org-social network models;
- $E$ is a finite set of coordinators or coordinator-groups connected to some external workflow-supported org-social network models;
- $\sigma = \sigma_1 \cup \sigma_o$ * Social Relationships: successors and predecessors */ where, $\sigma_o : C \rightarrow \varphi(C)$ is a multi-valued function mapping a performer to its sets of (immediate) candidate-successors, and $\sigma_1 : C \rightarrow \varphi(C)$ is a multi-valued function mapping a performer to its sets of (immediate) candidate-predecessors;
- $\psi = \psi_1 \cup \psi_o$ * Acquisition of Activities */ where, $\psi_1 : C \rightarrow \varphi(C)$ is a multi-valued function returning a bag of previously worked activities, $(K \subseteq A)$, on directed arcs, $(\sigma_1(o), o), o \in C$, from $\sigma_i(o)$ to $o$; and $\psi_o : C \rightarrow \varphi(C)$ is a multi-valued function returning a set of acquisition-activities, $(K \subseteq A)$, on directed arcs, $(o, \sigma_o(o)), o \in C$ from $o$ to $\sigma_o(o)$;

In principle, a workflow-supported org-social network model is graphically represented by a directed graph characterized by multiple-incoming arcs, multiple-outgoing arcs, cyclic, self-transitive, and multiple-activity associations on arcs. Additionally, it can be also transformed to an undirected graph for analyzing betweenness-centralities among the associated performers.

III. Algorithmic Formalisms for Betweenness Centrality Analysis

The most widely-used and basic methods that can be applied to analyze workflow-supported org-social networks are density, centrality, prestige, cohesiveness, structural equivalence, clustering, multidimensional scaling, and blockmodels. Out of them, we are particularly concerned about the workflow-performer’s prominence properties that are sought and quantified within a complete org-social network by summarizing the structural relations—typically centrality measures—among all nodes. In this section, we explicate a series of formulas and algorithmic formalisms that are needed to quantify the levels of prominences through the centrality method and its detailed analysis measures.

A. Implications

The primary use of the centrality method is to identify the important or prominent performers at both the individual and group levels of analysis. In general, the individual performer’s centrality reflects its greater visibility to the other performers, whereas the group-level indices of centralization assess the extent of an org-social network’s dispersion or inequality among all performer prominences. In the centrality aspect, the prominent performer has high involvement in many relations, regardless of whether sending or receiving ties in a corresponding workflow-supported org-social network.

The most widely-used centrality measures are degree, closeness, eigen-value, and betweenness. These measures vary in their applicability to non-directed and directed relations and differ at the individual performer and the whole group of a complete org-social network. We are particularly interested in quantitatively measuring the degree of betweenness centrality of a workflow-supported org-social network by extensively revising the well-known formulas in the conventional social network analysis arena. The analyzed measurements of betweenness centrality reflect how other performers control or mediate the relations between dyads that are not directly connected in a workflow-supported org-social network. The individual performer’s betweenness centrality gives the measurement of the extent to which other performers lie on the shortest distance between a pair of performers in the org-social network. Ultimately, the betweenness centrality measure gives a significant indicator of control over information exchange or resource flows within a workflow-supported org-social network. The implication of the algorithmic formalism to be deployed in this section aims to answer the following question:

- How much can a performer mediate (or control) the relations between other performers in enacting the associated workflow procedure?

B. SocioMatrix: Mathematical representation of a workflow-supported org-social network

The workflow-supported org-social network is mathematically represented by two classes—binary directed/nondirected SocioMatrix and valued directed/nondirected SocioMatrix—of socio-matrices so as to be analyzing cognitive organizational work-sharing structures. We use the socio-matrices to construct a sociogram that is a two-dimensional diagram for displaying the work-sharing relations among performers in a bounded workflow procedure. The term, directed, indicates directed relations or ties from the performer at the tail to the performer at the arrowhead; while the term, nondirected (no arrowheads), implies mutual relations. Likewise, when a directed/nondirected org-social network is transformed to a SocioMatrix, the term, binary, implies the most basic measurement, the presence or absence of a tie, which is a dichotomy indicated by binary values of 1 and 0, respectively; also socio-matrices may include nonbinary or valued cells, reflecting the intensity of relations or ties, such as frequency of contacts, tie strength, or magnitude of associations, and therefore the cell entries in the SocioMatrix can vary from 0 to the maximum level of dyadic interactions.
introduce the algorithms as followings, each of which produces binary directed SocioMatrix($Z^i_{in}([N, N])$, $Z^b_{out}([N, N])$), binary nondirected SocioMatrix($Z^b_{in}([N, N])$), valued directed SocioMatrix($Z^v_{in}([N, N])$, $Z^v_{out}([N, N])$), and valued nondirected SocioMatrix($Z^v_{in}([N, N])$), where $N$ is the number of performers, from a formally defined workflow-supported org-social network model.

**Binary Directed SocioMatrix Generation Algorithm**

Input A workflow-supported org-social network, $A = (\sigma, \psi, S, E)$;
Output Two symmetric binary SocioMatrices, $Z^b_{in}([N, N])$ and $Z^b_{out}([N, N])$, where $N$ is the number elements in the set of $C$ actors.

Begin Procedure 
Initialize all entries of $Z^b_{in}([N, N])$ to Zeros; 
Initialize all entries of $Z^b_{out}([N, N])$ to Zeros; 
For ($\forall o \in C$) Do 
Begin 
$^T$ The Incoming Relations of $Z^b_{in}([N, N])$ \nSet One To entries of $bZ_{in}(o)$, each member of $\sigma_i(o)$; \n$^T$ The Outgoing Relations of $Z^b_{out}([N, N])$ \nSet One To entries of $bZ_{out}(o)$, each member of $\sigma_i(o)$; 
End 
End Procedure

**Binary Non-Directed SocioMatrix Generation Algorithm**

Input A workflow-supported org-social network, $A = (\sigma, \psi, S, E)$;
Output A symmetric binary SocioMatrix, $Z^b([N, N])$, where $N$ is the number elements in the set of $C$ actors.

Begin Procedure 
Initialize all entries of $Z^b([N, N])$ to Zeros; 
For ($\forall o \in C$) Do 
Begin 
$^T$ Set the Incoming Relations to $Z^b([N, N])$ \nSet One To entries of $bZ_{in}(o)$, each member of $\sigma_i(o)$; \n$^T$ Set the Outgoing Relations to $Z^b([N, N])$ \nSet One To entries of $bZ_{out}(o)$, each member of $\sigma_i(o)$; 
End 
End Procedure

**Valued Directed SocioMatrix Generation Algorithm**

Input A workflow-supported org-social network, $A = (\sigma, \psi, S, E)$;
Output Two symmetric valued SocioMatrices, $Z^v_{in}([N, N])$ and $Z^v_{out}([N, N])$, where $N$ is the number elements in the set of $C$ actors.

Begin Procedure 
Initialize all entries of $Z^v_{in}([N, N])$ to Zeros; 
Initialize all entries of $Z^v_{out}([N, N])$ to Zeros; 
For ($\forall o \in C$) Do 
Begin 
$^T$ Add the Incoming Relations to $Z^v_{in}([N, N])$ \nAdd One To entries of $vZ_{in}(o)$, each member of $\sigma_i(o)$; \n$^T$ Add the Outgoing Relations to $Z^v_{out}([N, N])$ \nAdd One To entries of $vZ_{out}(o)$, each member of $\sigma_i(o)$; 
End 
End Procedure

**Valued Non-Directed SocioMatrix Generation Algorithm**

Input A workflow-supported org-social network, $A = (\sigma, \psi, S, E)$;
Output A symmetric valued SocioMatrix, $Z^v([N, N])$, where $N$ is the number elements in the set of $C$ actors.

Begin Procedure 
Initialize all entries of $Z^v([N, N])$ to Zeros; 
For ($\forall o \in C$) Do 
Begin 
$^T$ Add the Incoming Relations to $Z^v([N, N])$ \nAdd One To entries of $vZ_{in}(o)$, each member of $\sigma_i(o)$; \n$^T$ Add the Outgoing Relations to $Z^v([N, N])$ \nAdd One To entries of $vZ_{out}(o)$, each member of $\sigma_i(o)$; 
End 
End Procedure

C. Individual Betweenness Centralization Analysis

Based upon the socio-matrices, we are able to measure the betweenness centralization by applying the formulas given in [7]. Through the betweenness centrality method and its measurements we can obtain a reasonable level of analysis results, which is enough to answer to the issued question stated in the beginning of the section. The betweenness centrality measures can be applied to the individual performer (individual betweenness centrality) as well as the group of performers (group betweenness centrality) in a workflow-supported org-social network. In this subsection, we develop an individual betweenness centrality analysis algorithm based upon the formulas of the individual betweenness centralization.

1) Formula: As well-stated in [7], an individual performer’s betweenness centrality is based on an essential function of $G_{jk}$ denoting the number of possible geodesic paths (shortest distances) between the two performer-nodes $j$ and $k$. $G_{jk}(N_i)$ gives the number of geodesics containing the performer-node $i$ out of $G_{jk}$. So, the proportion dividing $G_{jk}(N_i)$ by $G_{jk}$ gives the ratio of geodesic paths connecting $j$ and $k$ through performer-node $i$.

At this moment, we would emphasize that we do apply not the concept of geodesic path but the concept of work-transferring path to the formulas on workflow-supported org-social networks. The geodesic path implies the shortest distance between two nodes, which is inappropriate for the workflow-supported org-social networks. The formulas use all the possible paths rather than only the shortest paths in formalizing the betweenness centralization. At last, the work-transferring path implies a possible line connecting an arbitrary dyad (two performer-nodes) on a workflow-supported org-social network. Performer $j$ has to go through performer $i$ to collaborate with performer $k$. Performer-node $i$ has responsibility or control over the work-content and shift-timing in transmitting work-opportunity between performer-nodes $j$ and $k$. The more often that performer-node $i$ is located on the work-transferring path between numerous dyads, the higher performer-node $i$ has potential to control work-collaborating interactions.

The conceptual implication of the individual betweenness centrality refers to how much a performer can control or mediate the work-transferring between numerous dyads. So, let’s redefine the meanings of the functions, $G_{jk}$ and $G_{jk}(N_i)$. An individual performer’s betweenness centrality is based on an essential function of $G_{jk}$ denoting the number of possible work-transferring paths between the two performer-nodes $j$ and $k$. $G_{jk}(N_i)$ gives the number of work-transferring paths containing the performer-node $i$ out of $G_{jk}$. The proportion dividing $G_{jk}(N_i)$ by $G_{jk}$ gives the ratio of work-transferring paths connecting $j$ and $k$ through performer-node $i$. Conclusively, for a binary nondirected workflow-supported org-social network with $q$ performers, the individual closeness centrality measures are computed by summing of the proportions of the geodesic paths between all the dyads in which performer-node $i$ involved, as shown in the following formulas:

- The Index of Individual Betweenness Centrality

$$C_B(N_i) = \sum_{j<k} \frac{G_{jk}(N_i)}{G_{jk}}$$

(1)

- The Standardized Index of Individual Betweenness Centrality

$$C_B^S(N_i) = \frac{C_B(N_i) \times 2}{(q-1)(q-2)}$$

(2)
As you see, the measures computed from the formula (1)[7] can be 0.0 if performer-node $i$ is not involved in any work-transferring paths for all the dyads, whereas they can be $(q-1)(q-2)$ if performer-node $i$ falls on every work-transferring path for all the dyads assuming only when each dyad (pair of $j$ and $k$) has no more than one work-transferring path.

In order to control the size of the org-social network, it is necessary for the individual index to be normalized between 0.0 and 1.0 so as to allow meaningful comparisons of performers across different org-social networks. The formula (2)[7] is for standardizing the index of individual betweenness centrality by multiplying the maximum theoretical value of $(q-1)^2(q-2)$.

Note that the maximum theoretical value ought to be larger than $(q-1)^2(q-2)$ if a dyad has more than one work-transferring path.

2) Algorithm: We develop an algorithm concretizing the index of individual betweenness centrality of the formula 1. The algorithm is able to calculate the basic functions of $G_{jk}$ and $G_{jk}(N_i)$ for a specific performer-node, $o_i$, through the procedure name of $iC_B$Measurement with a recursive subroutine of $iGeodesicPath$. The algorithm described in this section uses a binary nondirected socio-matrix as input. However, we can extensively apply this algorithm to the remainder of other types of socio-matrices.

The Individual Betweenness Centrality Analysis Algorithm:

```
Procedure Name: $iC_B$Measurement
Input Performer, $o_i$;
Input A Binary Nondirected SocioMatrix, $Z^B[N, N]$;
Output $o_i$’s Individual Betweenness Centrality, $C_B(N_i)$;
Begin Procedure
Initialize $G[N, N]$; $iC_B[N, N]$; $iGeodesicPath[N, N]$;
For($\forall o_i \in C$)
    $o_m \leftarrow o_i$;
    For($\forall o_j \in C, o_j \neq o_k \wedge j < k$)
        $o_k \leftarrow o_j$;
        $iC_B[N, N] \leftarrow iGeodesicPath(o_k)$;
        Ref:
        $\sum_{j=1}^{N} \sum_{k=j}^{N} G[o_j, o_k] \leftarrow iC_B[N, N]$;
End Procedure
```

D. Group Betweenness Centralization Analysis

Based upon the formula and its implemented algorithm expatiated in the previous subsection, we are able to analyze group level betweenness centralization in a workflow-supported org-social network, too. The group level betweenness centralization quantitatively measures the overall tendency of betweenness across performers in an org-social network, and gives the extent to which performers in a given org-social network differ in their betweenness centralities, as well. The index of group betweenness centrality is computed as followings:

- The Index of Group Betweenness Centrality[7]

\[
C_B = \sum_{i=1}^{g} \frac{[C_B(N^*) - C_B(N_i)]}{(q-1)^2(q-2)}
\]

In the formula of (3)[7], $C_B(N^*)$ denotes the highest possible value, and so the numerator is summing the differences in betweenness centralities for the performer with the highest value and every other performer. The denominator indicates the maximum theoretical possible value of betweenness centralities for all performer-nodes in an org-social network with $g$ performers, which is $(q-1)^2(q-2)$.

Conclusively, the index of group betweenness centrality measurement in a workflow-supported org-social network may

\[^3\]Excluding performer-node $i$, the number of work-transferring paths among the $g-1$ performer-nodes is $C^2_{g-1} = \frac{(q-1)(q-2)}{2} = \frac{(q-1)^2(q-2)}{2}$.
take values between 0.0 and 1.0. When a single dominant performer is placed on all work-transferring paths, it reaches 1.0. In contrast, when every performer has the same betweenness centrality, it reaches 0.0 because the numerator becomes zero. Thus, the closer the group betweenness centralization approaches to 1.0, the more unequally distributed betweenness centrality within the corresponding org-social network is.

IV. Conclusion

In this paper, we suggested a series of formulas for quantifying the knowledge of work-mediating behaviors among workflow-supported people by revising the betweenness centrality analysis techniques[7] of social networks. Based upon the formulas, we devised an automatic analysis algorithm to calculate an individual performer’s betweenness centralization measurements on a workflow-supported org-social network model. The algorithm proposed in this paper ought to be an impeccable solution for developing the automatic analysis and visualization functionalities for workflow-supported org-social networking knowledge management systems. Likewise, as a future work, we need to develop the remainder centrality analysis techniques, like eigen-value centralities, to be applied to workflow-supported org-social network models.

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