Ergodic Capacity Analysis of OFDM-based NB-PLC Systems

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Abstract—Power line communication (PLC) has been proposed as a crucial part of smart grid applications. This is due to its capacity to use existing power-grid infrastructure to provide cost-efficient data transmissions. However, the communication performance might be degraded by channel attenuation and high unpredictable noise levels. Consequently, the performance analysis of PLC is of high importance. In this paper, we investigated and evaluated the performance of orthogonal frequency division multiplexing (OFDM)-based narrow-band (NB)-PLC systems in terms of ergodic capacity. Mathematical tools were used to derive the expression of the corresponding average ergodic capacity (AEC). Furthermore, field measurements were conducted to confirm the theoretical results and assess the NB-PLC system’s performance. The theoretical and field measurement results confirm the impact of the selected channel on the performance of the considered OFDM-based NB-PLC systems. Notably, with respect to the conducted field measurements in the NB-PLC Federal Communications Commission (FCC) band, it is showed that some frequency sub-bands enjoy a higher ergodic capacity with respect to rest of the frequency spectrum.

Index Terms—PLC, Field Measurements, Average Ergodic capacity, NB-PLC, OFDM, Performance Analysis, Smart Grid.

I. INTRODUCTION

POW ER LINE COMMUNICATION (PLC) is currently one of the most popular solutions for smart grid applications due to its ability to reuse existing power-grid infrastructure. PLC utilizes the electric power grid as the communication medium, reducing infrastructure costs and, therefore, resulting in cost-efficient transmissions. Using the power cables as a transmission medium also provides additional security as the use of those cables is physically limited [1]. This significantly increases the security of existing and vulnerable electrical grids [2] as it is notably more difficult for an unauthorized person to gain access to a power cable that runs within a wall or that is buried underground. Physical layer security schemes based on the channel features of PLC may also be used to provide another layer of protection [3]–[5].

Based on the used frequency band, PLC can be divided into three categories: ultra narrow-band (UNB)-PLC, narrow-band (NB)-PLC, and broadband (BB)-PLC [6], [7]. The number of installed PLC devices has grown exponentially in the recent years. In addition to the traditional uses of PLC in home area networks and building automation, new applications, particularly those related to power utility distribution networks, have extensively leveraged this technology for their smart grid deployments. In 2018, 99+ million electrical smart meters were put in place in the European Union, with the prediction that 220+ million would be installed by 2024 [8], [9]. Many of these projects employ NB-PLC as the primary means of communication [10], [11].

NB-PLC uses the frequency range from 3 to 500 kHz, being PRIME, G3-PLC, G.hn, LF SGPLC, and MF SGPLC the most representative standards operating in this band [1]. The field measurements in this paper were performed in the FCC band and segmented into eight sub-bands for analysis following the PRIME 1.4 technology standard [12]. PRIME, an acronym for PoweRline Intelligent Metering Evolution, is a worldwide PLC technology for smart metering and real-time energy management applications [13]. This technology can be deployed to offer intelligent monitoring and control capabilities; for example, the operators can monitor electricity consumption throughout the grid in real-time, implement variable tariff schedules, and set limits on electricity consumption to better manage peak loads [13]. Consequently, variable tariff schedules and real-time visibility into electricity consumption can encourage consumers to reduce electricity consumption during peak usage times [13]–[15]. PRIME technology has been developed to overcome the challenges of PLC in harsh communication environments. Its conception is based on telecommunications architecture, that supports orthogonal frequency division multiplexing (OFDM) transmission technique, low/medium voltage smart metering, and the integration of renewable and home area network functionalities. The PRIME standard (Recommendation), ITU-T G.9904 [16], was published by ITU-T in October 2012. Subsequently, PRIME Alliance published two versions of the PRIME specification (v.1.3.6 and v.1.4) to detail the ITU-T G.9904 technical specifications. The first version, PRIME v.1.3.6, uses the frequency bands from 42 kHz to 89 kHz and the modulations D8PSK, DQPSK, and D8PSK to offer a peak data rate of 128 kbps [17]. The extended version of the PRIME specification was released to adapt to new regulations and markets. It presents new features, such as...
novel robust modes, a new PHY frame type, and the use of the entire frequency band from 42 to 471 kHz, which results in multiplying the original bandwidth by eight to reach the peak data rate of 1 Mbps [12], [18].

Similar to most communication systems, NB-PLC suffers from channel attenuation and additive noise as major transmission problems [19]–[21]. Channel attenuation is directly proportional to the frequency and the power line (PL) length, increasing in tandem with the values of these two factors. When PLC signals are severely attenuated, the noise will have a significant impact on system performance [22]–[24]. Consequently, the performance analysis of NB-PLC systems, in different scenarios and locations, is of high importance.

In the literature, various related works have focused on the performance analysis of the systems. In [25], the authors have evaluated the performance of a given NB-PLC system in terms of the average bit error rate (BER). Closed-form average BER expressions have been derived in [26] to investigate the performance of different PLC physical-layer system models. By considering background and impulsive noises, the authors in [27] have detailed the derivation of the corresponding probability density function (PDF) and the BER expressions of a binary phase-shift keying modulated signal. In [28], field measurements have been used to estimate the data bit rate used to evaluate an example of a PLC system. However, none of the aforementioned related works have used both practical and theoretical analyses to evaluate the asymmetry of NB-PLC systems.

In our previous work [29], we have presented a performance analysis of single-carrier PLC systems in terms of successful transmission probability (STP) and on field measurements to assess the performance of a single-carrier-based PLC system. In addition to the STP metric, the average ergodic capacity (AEC) is another important metric which refers to the maximum rate at which a given communication can be achieved for a given frequency unit, which is expressed in bps Hz$^{-1}$.

In this paper, we investigated the performance of an OFDM-based NB-PLC system in terms of the AEC, using mathematical derivation tools as well as field-measurement-based simulations. The results have confirmed the accuracy of the derived AEC expression, which can be used to ensure optimal deployment of new PLC systems.

The remainder of this paper is organized as follows. Noise characteristics and the derivation of the expression for AEC are detailed in Section II. The field measurements are presented in Section III, and the results and interpretations are set out in Section IV. Finally, conclusions are drawn in Section V.

## II. MODELLING AND PERFORMANCE ANALYSIS

### A. PLC Noise Characteristics

In OFDM-based PLC, the noise is classified into three categories: narrow-band noise, colored background noise, and impulsive noise [30], [31]. Narrow-band noise is produced by amateur radio, wireless communication and other broadcasting systems. It consists of time-variant modulated amplitude sinusoidal signals. Coloured background noise is a superposition of different noise sources with a low power spectral density (PSD). Plugged electrical appliances generate impulsive noise categorized into three types [32]: periodic synchronous impulsive noise, periodic asynchronous impulsive noise, and non-periodic asynchronous impulsive noise. Periodic synchronous impulsive noise consists of synchronous impulses with the main PL frequency and is generated by the rectifier diodes in power supplies (e.g., power converters) that operate synchronously with the main frequency. Periodic asynchronous impulsive noise is produced by switching power supplies on the network and AC/DC power converters. Non-periodic asynchronous impulsive noise generated by switching transients in the network is considered the major impairment to PLC [31]. These impulses can be up to 105 times stronger than the background noise [32], making them the principal cause of occurrences of error in digital communications over PLC networks [30], [32]–[34] Middleton Class A distributions are commonly used to model the most significant PLC noises [30], [33], [34]. To obtain tractable analytical results, the Middleton Class A distributions can be approximated by a Gaussian distribution of a large impulsive index [33]. In this case, the real and imaginary components of the complex noise signal can be assumed to be Gaussian random variables with a zero mean and PLC channel-dependent variance. Accordingly, the PDF of the instantaneous noise power of sub-carrier index $k$, is given by the equation:

$$f_k(N_k) = \frac{1}{N_k} \exp \left(-\frac{N_k}{N_k} \right).$$  (1)

where $N_k$ denotes the average of $N_k$.

### B. Average Ergodic Capacity expression derivation

In this section, we derived the expression for the AEC to evaluate the performance of the considered OFDM-based NB-PLC system. Mathematically, the AEC of a given subcarrier index $k$ can be written as:

$$AEC_k = \int_0^\infty \int_0^\infty f_k(y) g_k(x) \log_2 \left(1 + \gamma_k(x, y)\right) \, dx \, dy,$$  (2)

where, $\gamma_k(x, y)$ is the corresponding instantaneous SNR. Here, $x$ presents the instantaneous channel gain of sub-carrier index $k$, $H_k$, with PDF $g_k(x)$, and $y$ presents the instantaneous noise power of sub-carrier index $k$, $N_k$.

The general $\gamma_k$ expression can be written as follows:

$$\gamma_k = \frac{P_r H_k}{N_k}.$$  (3)

Based on [19]–[21], the considered PLC channels are assumed to be Rayleigh fading channels. Consequently, the PDF expression of $H_k$ is given by [19]–[21]

$$g_k(H_k) = \frac{1}{A_k(f_{k, l})} \exp \left(-\frac{H_k}{A_k(f_{k, l})}\right),$$  (4)
where \( A_k(f_k, l) \) is the mean of \( H_k \), which presents the average frequency-distance dependent PLC channel attenuation, \( f_k \) is the subcarrier frequency index \( k \), and \( l \) is the PL length.

Based on [21]–[24], \( A_k(f_k, l) \) can be expressed as follows:

\[
A_k(f_k, l) = \exp \left( -2 \left[ a_0 + a_1 f_k^{a_2} \right] l \right),
\]

(5)

where, \( a_0, a_1, \) and \( a_2 \) are attenuation parameters.

Based on (2), (3), and (4), \( AEC_k \) can be rewritten as follows:

\[
AEC_k = \frac{1}{A_k(f_k, l)} \int_0^\infty f_k(y) \times \int_0^\infty \exp \left( -\frac{y}{A_k(f_k, l)} \right) \log_2 \left( 1 + \frac{P_T x}{y} \right) \, dx \, dy,
\]

(6)

By using the change in variables; \( z = 1 + \frac{P_T x}{y} \), (6) yields to

\[
AEC_k = \frac{1}{\ln(2)} A_k(f_k, l) P_T \int_0^\infty y f_k(y) \exp \left( \frac{y}{P_T A_k(f_k, l)} \right) \times \int_1^\infty \exp \left( -\frac{y}{P_T A_k(f_k, l)} \right) \ln(z) \, dz \, dy.
\]

(7)

Now, by using [Eq. (4.331.2), [35]], the integrals in (7) with respect to \( z \) can be evaluated as follows:

\[
AEC_k = -\frac{1}{\ln(2)} \int_0^\infty f_k(y) \exp \left( \frac{y}{P_T A_k(f_k, l)} \right) \times \text{Ei} \left( -\frac{y}{P_T A_k(f_k, l)} \right) dy.
\]

(8)

where, \( \text{Ei} \) is the exponential integral function, which is expressed as [Eq. (8.211.1), [35]]

\[
\text{Ei}(x) = -\int_{-x}^\infty \exp(-t) \, t^{-1} dt
\]

(9)

By using the PDF expression \( f_k \), (8) becomes:

\[
AEC_k = -\frac{1}{\ln(2) N_k} \int_0^\infty \exp \left( -\left[ \frac{1}{N_k} - \frac{1}{P_T A_k(f_k, l)} \right] y \right) \times \text{Ei} \left( -\frac{y}{P_T A_k(f_k, l)} \right) dy.
\]

(10)

After using the change in variables; \( x = \frac{P_T A_k(f_k, l)}{y} \), the \( AEC_k \) expression can be written as follows:

\[
AEC_k = -\frac{P_T A_k(f_k, l)}{\ln(2) N_k} \times \int_0^\infty \text{Ei}(-x) \exp \left( -\left[ \frac{P_T A_k(f_k, l)}{N_k} - 1 \right] x \right) \, dx.
\]

(11)

Finally, by using [Eq. (6.224.1), [35]], and after some simplification, the integration in (11) is evaluated and the \( AEC_k \) final expression is given by follows:

\[
AEC_k = \frac{x_k}{(x_k - 1) \log_2(x_k)}.
\]

(12)

with,

\[
x_k = \frac{P_T}{N_k} \exp \left( -2 \left[ a_0 + a_1 f_k^{a_2} \right] l \right)
\]

(13)

As shown in (12), the AEC decreases with increasing values of the average noise \( N_k \). In this case, it converges to zero as \( N_k \) increases to a very large values, which is expected. On the other side, the AEC increases with decreasing values of \( N_k \), and it converges to large value with logarithmic growth, with respect to increasing values of \( N_k \).

### III. Field Measurements

The field measurements were performed using the PL360 modem evaluation kit of Microchip [36], [37]. The PL360 is a multi-protocol PLC modem designed with a flexible architecture that allows for the implementation of standard and customized PLC solutions [36]. It includes a SAM4CMS16C ARM Cortex-M4 microcontroller, which provides a full-featured platform to develop a complete communication system over PLC technology [37]. The PHY tester tool, available in the ATPL 360-EK, was used to evaluate the physical layer characteristics of the FCC band. The PL360 was connected to a Raspberry Pi running the measurement scripts.

The measurements of the noise were performed in the NB-PLC FCC band from 42 to 472 kHz, and they were segmented into eight sub-bands following version 1.4 of the PRIME technology standard [12]. The measurement period lasted one complete week, with multiple measurement cycles.

The field test was conducted by connecting the measurement set in the meter box beside the metrology meter of a standalone villa in Doha. The area, composed of a 1,600 kVA oil-filled transformer feeding a set of villas and buildings, corresponded to the typical semi-urban topology.

### IV. Results and Discussion

In this section, we present and investigate the analytical and field-measurement-based simulation results of the PLC AEC between two given nodes. Without loss of generality, the used simulation parameters were: \( a_0 = 8.4 \times 10^{-3}, \ a_1 = 3 \times 10^{-9} \) m\(^{-1} \), \( a_2 = 1 \) [22], \( P_T = 126 \) dBuV, and \( l = 300 \) m. The average noise power per subcarrier, \( N_k \), was evaluated based on the conducted field measurements.

Fig. 1 shows the time-averaged noise levels of all the performed measurements for the eight channels at the considered receiver during the day and night periods to assess the performances in different time periods. As shown in Fig. 1, channel 1 has the highest level of noise power when compared to the other channels, corresponding to the results of previous
In Figs. 2, and 3, we present the variation of the AEC during the day and night periods for channel 1, and channel 6, respectively. The presented results confirm the accuracy of the derived expression for the AEC, where the theoretical curve fits well with the simulated one for both periods. In addition, the AEC during the daytime was shown to be higher than that of the nighttime. This is due to the fact that, as shown in Fig. 1, the nighttime period is affected by a higher noise level when compared to the situation in the daytime period.

The same results were shown for the other channels, as presented in Fig. 4, where the variations in the AEC, during the day and night periods, for all channels are presented.

V. CONCLUSION

The performance of OFDM-based NB-PLC systems, in terms of AEC, has been investigated in this paper. We first reviewed the noise characteristics of NB-PLC and subsequently introduced and derived the expression for the AEC. Based on long-term measurements in the field, the performed simulations confirm the accuracy of the derived expression for the different tested periods. Some frequency sub-bands have shown a higher ergodic capacity than others, which confirms the effects of channel selection on OFDM-based NB-PLC systems performance.
In our future work we will propose and evaluate new techniques to enhance the OFDM-based NB-PLC systems.

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